1363

B.Sc. (Hons.) Microbiology /

Biochemistry / Bio-medical / II Sem.   **A**

Paper – MACT-303

Mathematics & Statistics

(Admission of 2010 and onwords)

Time : 3 Hours   **Maximum Marks : 75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are **three** Sections in this question paper.

Attempt any **two** questions from each Section.

Students are allowed to use Simple calculators.

Use of log tables and normal tables are allowed.

SECTION – I

1. (a) Let L_1 and L_2 be two straight lines in a plane. Then write down all possible values for $L_1 \cap L_2$.   4

(b) Evaluate

(i) $\lim\limits_{x \to \infty} \left(1 - \dfrac{2}{x}\right)^x$

(ii) $\lim\limits_{x \to 9} \left(\dfrac{\sqrt{x} - 3}{x - 9}\right)$

(iii) $\lim\limits_{x \to 0} \dfrac{\sin 5x}{\sin 6x}$ 6

(c) The pressure P and volume V of a given mass of gas are connected by the relation PV = 800, when the temperature is held fixed.

(i) Find the average rate of change of P as V increases from 200 to 250.

(ii) Show that the rate of change of V with respect to P is inversely proportional to the square of P.   5

2. (a) If $y = Ae^{-kt} \cos (pt + c)$, then show that

$$\dfrac{d^2y}{dt^2} + 2k \dfrac{dy}{dt} + (k^2 + p^2)\, y = 0.$$ 5

(b) Integrate any **two** :   5

(i) $\displaystyle\int e^x \sin 2x \, dx$

(ii) $\displaystyle\int \dfrac{e^x (1 + x)}{\cos^2 (xe^x)} dx$

(iii) $\displaystyle\int \left(x + \dfrac{1}{x}\right)\left(x^2 + \dfrac{1}{x^2}\right) dx$

(c) A ball is dropped from a height of 10 m. Each time it strikes the ground it bounces vertically to a height that is $\frac{3}{4}$ of the preceeding height. Find the total distance the ball will travel if it is assumed to bounce often. 5

3. (a) A parking lot of C.P. charges Rs. 10 for 1st 2 hours or less and Rs. 5 for each succeeding half hour upto maximum of Rs. 30.

   (i) Sketch a graph for the cost of parking as a function of time.

   (ii) Discuss its discontinuity. 5

(b) A cell multiples its number by dividing itself as per the following chart :

Time (in seconds)	Cell Division Chain	Total no. of cells
t = 0		1
t = 1		2
t = 2		4
t = 3		8

Calculate the total number of cells when t = 10. 5

(c) Differentiate any two functions with respect to x :

(i) $y = \dfrac{\cos x - \sin x}{\cos x + \sin x}$

(ii) $y = e^{ax} \cos (bx + c)$

(iii) $y = \dfrac{\sqrt{a} + \sqrt{x}}{\sqrt{a} - \sqrt{x}}$ 5

SECTION – II

4. (a) The following two matrices are taken from genetics :

$$T = \begin{bmatrix} p & q & 0 \\ \dfrac{1}{2}p & \dfrac{1}{2} & \dfrac{1}{2}q \\ 0 & p & q \end{bmatrix} \quad O = \begin{bmatrix} p^2 & 2pq & q^2 \\ p^2 & 2pq & q^2 \\ p^2 & 2pq & q^2 \end{bmatrix}$$

where $p + q = 1$. Verify that $TO = O$.   5

(b) If $A = \begin{bmatrix} 1 & 3 & 2 \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 5 & 0 \\ 0 & 1 & 5 \\ 2 & 1 & 3 \end{bmatrix}$,

find

(i) $(A + B)^T$

(ii) $A^T + B^T$

Further check if $(A + B)^T = A^T + B^T$.   5

5. (a) Find the matrix X such that AX = B, where

$$A = \begin{bmatrix} 3 & 4 \\ 6 & 2 \end{bmatrix}, B = \begin{bmatrix} 11 & 9 \\ 10 & 0 \end{bmatrix}$$ 5

   (b) Find the image of a point (3, −2) under the following transformation using matrix multiplication :

   (i) Reflection in X-axis

   (ii) Rotation through an angle 30° in the counter clockwise direction. 5

6. (a) If $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}, B = \begin{bmatrix} 2 & 5 \\ 0 & 4 \end{bmatrix},$

   $C = \begin{bmatrix} -1 & -2 \\ 3 & 0 \end{bmatrix}$, find out A · (B + C) in two ways according to distributive laws. 5

   (b) What matrix has the effect of rotating every vector through 90° and then projecting the result on X-axis ? 5

SECTION – III

7. (a) The mean of the marks obtained in an examination by a group of 100 students was found to be 49.46. The mean of the marks obtained in the same examination by another group of 200 students was 52.32. Find the mean of the marks obtained by both the groups taken together. 4

(b) The following frequency table gives the ages of a group of 50 children in a party. Find the standard deviation of the distribution :   4 ½

Age (In years)	10 – 15	15 – 20	20 – 25	25 – 30	30 – 35
Frequency	15	14	10	7	4

(c) Each of the five rats are injected with the certain Poisson and the number of rats dying within 24 hours is observed. Suppose that the .probability of one or more rats dying is 7/10 and the probability of four or less rats dying is 3/10. Find the probability that   4

(i) 5 rats die

(ii) No rat die ·

8. (a) Two flower seeds are randomly selected from a bag containing 10 seeds for red flowers and 5 seeds for white flowers. What is the probability that   3

(i) Both result in white flowers

(ii) One of each colour is selected.

(b) Find the straight line to the data :   5 ½

x	0	1	2	3	4
y	1	1.8	3.3	4.5	6.3

(c) Two random variables have least square regression lines $3x + 2y = 26$ and $6x + y = 31$. Find the mean values and coefficient of correlation. 4

9. (a) For a random sample of 10 pigs fed on a diet A, the increase in weight in a certain period were 10, 6, 16, 17, 13, 12, 8, 14, 15, 9 lbs. For another random sample of 12 pigs fed on diet B, the increase during the same period were 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 lbs.

Show that the mean increase of 12 and 15 lbs are not significantly different given that

$P(|t| \geq 2.093) = 0.05$ for 20 degree of freedom. 6½

(b) The length of a certain species of fish are normally distributed with mean 60 cm. If 1.22% of the fish are less than 51, determine the standard deviation. 6