[This question paper contains 5 printed pages.]

1211 *Your Roll No.* ....

B.Sc. (Hons.)/I **A**

PHYSICS – Paper I

(Mathematical Physics – I)

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Five** *questions in all.*
Question No. **1** *is compulsory.*
Attempt **one** *question from each Section.*

1. Do any **four** parts :

(a) Define polar and axial vectors. Give one example of each.

(b) By calculating the Wronskian of the functions x^n, $x^n \log x$, check whether the functions are linearly dependent or independent.

(c) Write down the Euler Lagrange equation.

(d) Find $\frac{d^2}{dt^2}\left(\vec{V}\cdot\frac{d\vec{V}}{dt}\times\frac{d^2\vec{V}}{dt^2}\right)$.

(e) State the Dirichlet conditions for a Fourier series expansion.

(f) What is the significance of Precision constant for a given data. (1½×4)

SECTION A

2. (a) Find the directional derivative of

$$\phi = 4xz^3 - 3x^2y^2z \text{ at } (2, -1, 2)$$

in the direction $2\hat{i} - 3\hat{j} + 6\hat{k}$. (2)

(b) Verify Stokes theorem for

$\vec{F} = xz\hat{i} - y\hat{j} + x^2y\hat{k}$ where S is the surface of the region bounded by $x = 0$, $y = 0$, $z = 0$, $2x + y + 2z = 8$ which is not included in x–z plane. (4)

(c) Evaluate

$$\vec{\nabla} \cdot \left\{ r\, \vec{\nabla}\left(\frac{1}{r^3}\right) \right\}.$$

where $r = (x^2 + y^2 + z^2)^{1/2}$ (2)

3. (a) If $\vec{F} = (5xy - 6x^2)\hat{i} + (2y - 4x)\hat{j}$

evaluate $\int_C \vec{F} \cdot \vec{dr}$ along the curve C in x–y plane : $y = x^3$ from the point (1, 1) to (2, 8). (2)

(b) Prove that curl of a vector is always solenoidal in nature. (2)

(c) State and prove Gauss Divergence Theorem. (4)

SECTION B

4. (a) Starting from the first principle, derive an expression for divergence of a vector in orthogonal curvilinear coordinates. (3)

(b) Find the components of a vector :

$$\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$$

in cylindrical coordinate system. (3)

(c) Using Lagrange's method of undetermined multipliers, find a point in the plane

$x + 2y + 3z = 13$ nearest to point (1, 1, 1) (2)

5. (a) Prove that the shortest distance between two points in a plane is a straight line. (3)

(b) Find the Jacobian of transformation (2)

$$J\left(\frac{x, y, z}{u, v, w}\right)$$

if $x = u^2 + 2$

$y = u + v$

(c) Evaluate

$$\oiint_S (x^3\,dy\,dz + y^3\,dz\,dx + z^3\,dx\,dy)$$

$\forall$ S is the surface of the sphere $x^2 + y^2 + z^2 = 1$. (3)

SECTION C

6. Solve the following differential equations :

(a) $x \cos x \dfrac{dy}{dx} + y(x \sin x + \cos x) = 1$ (2)

(b) $\dfrac{d^2y}{dx^2} + 9y = \sec 3x$

by using method of variation of parameters. (2½)

(c) $x^2 \dfrac{d^2y}{dx^2} + 3x\dfrac{dy}{dx} + y = (1-x)^2$. (3½)

7. (a) Solve the differential equation by the method of undetermined coefficients :

$$\frac{d^2y}{dx^2} + \frac{2dy}{dx} + y = e^{-x}$$ (3)

(b) Find the solution of

$\dfrac{d^2y}{dx^2} + 3\dfrac{dy}{dx} + 2y = e^{-x}$ that satisfies the initial conditions $y = 0$, $\dfrac{dy}{dx} = 0$ at $x = 0$. (2)

(c) Solve the differential equation :–

$(D^2 + 1)y = \cos x + e^x \sin x$ (3)

SECTION D

8. (a) Expand the function

$f(x) = x + x^2$ in fourier series in the interval $(-\pi, \pi)$

and hence deduce that

$$1 + \frac{1}{2^2} + \frac{1}{3^2} + \cdots = \frac{\pi^2}{6}$$ (5)

(b) State and prove Normal law of Errors. (3)

9. (a) Expand f(x) as a sine series when

$$f(x) = \begin{cases} x & 0 < x < \pi/2 \\ \pi - x & \pi/2 < x < \pi \end{cases}$$ (4)

(b) By using the principle of least squares, find the equation of best fit straight line in following data :

x →	0	5	10	15
y →	12	15	17	22

(4)