This question paper contains 4+2 printed pages]

Your Roll No................

5701

B.Sc. (Hons.) PHYSICS/I Sem. B

Paper—PHHT-101

(Mathematical Physics)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all

including Q. No. 1 which is compulsory.

1. Do any *five* parts : 5×3=15

(*a*) Find the unit tangent vector at the point $t = 2$ on the curve :

$$x = t - \frac{t^3}{3}, y = t^2, z = t + \frac{t^3}{3}.$$

(*b*) Determine $\vec{\nabla} \cdot \left(\frac{\vec{r}}{r^n}\right)$, $n > 0$ and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(*c*) If $u = \frac{y^2}{2x}$ and $v = \frac{x^2 + y^2}{2x}$, find the Jacobian $J\left(\frac{u, v}{x, y}\right)$.

(*d*) Consider a periodic function $f(x)$ of period 2π :

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

Plot $f(x)$, locate its discontinuities and find the value of $f(x)$ at $x = 0$.

(*e*) Define beta function and find the value of $\beta\left(\frac{3}{2}, 2\right)$.

(*f*) State Normal Law of Errors.

(*g*) If $\vec{B} = \vec{\nabla} \times \vec{A}$, then prove that $\iint_S \vec{B} \cdot \hat{n} \, dS = 0$ for any closed surface S.

(*h*) Evaluate $\iint_R \sqrt{x^2 + y^2} \, dx \, dy$, where R is the region defined by $x^2 + y^2 = a^2$.

2. (*a*) Prove that : 4

$$\vec{\nabla} \cdot (\phi \vec{A}) = (\vec{\nabla} \phi) \cdot \vec{A} + \phi(\vec{\nabla} \cdot \vec{A}).$$

(*b*) Evaluate : 6

$$\vec{\nabla} \left[\vec{r} \cdot \vec{\nabla} \left(\frac{1}{r^3} \right) \right].$$

(*c*) Show that : 5

$$\vec{A} = (6xy + z^3)\,\hat{i} + (3x^2 - z)\,\hat{j} + (3xz^2 - y)\hat{k}$$

is irrotational. Find ϕ such that $\vec{A} = \vec{\nabla} \phi$.

3. (*a*) State and prove Gauss' divergence theorem. 3,7

(*b*) Evaluate $\oint_C (2x + y^2)\,dx + (3y - 4x)dy$, where C is the closed curve shown in Fig. 1 : 5

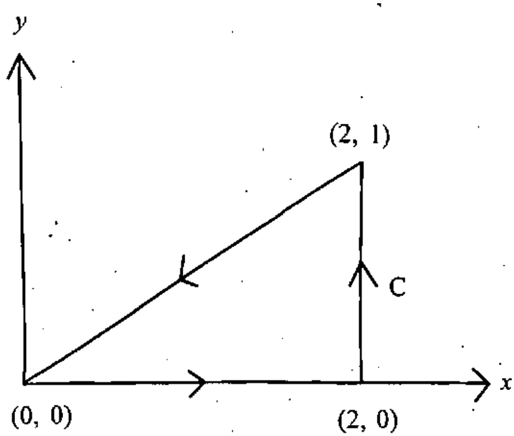

Fig. 1

4. (*a*) Derive an expression for the gradient of a scalar function ψ in orthogonal curvilinear co-ordinates and hence derive the expression of curl of a vector field. Express them in spherical co-ordinate system. 3,5,2

(*b*) Evaluate $\iiint_V (y^2 + z^2)\, dV$, where V is the volume enclosed by the cylinder : 5

$$x^2 + y^2 = a^2, 0 \le z \le b.$$

5. (*a*) Verify Stokes theorem for

$$\vec{A} = (y - z + 2)\hat{i} + (yz + 4)\hat{j} - xz\hat{k}$$

over 'S', the surface of the cube $x = 0$, $y = 0$, $z = 0$, $x = 2$, $y = 2$, $z = 2$ above the xy-plane. 10

(*b*) Prove that : 5

$$\iiint_V (\phi\nabla^2\psi - \psi\nabla^2\phi)\, dV = \iint_S (\phi\vec{\nabla}\psi - \psi\vec{\nabla}\phi).\, dS$$

6. (*a*) Prove that : 7

$$\Gamma(n)\Gamma\left(n+\frac{1}{2}\right)=\frac{\sqrt{\pi}\,\Gamma(2n)}{2^{2n-1}}.$$

(*b*) Prove that : 5

$$\beta(m, n)=\frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

(*c*) Evaluate : 3

$$\int_0^\infty \frac{y^2 dy}{y^4+1}.$$

7. (*a*) The length of cylinder when measured yields the following values (in cm) : 6

4.19, 4.21, 4.17, 4.20, 4.18, 4.23 and 4.22

Find the mean length and its standard error.

(*b*) The radius r of a cylinder is given as (2.1 ± 0.1) cm and height h as (6.4 ± 0.2) cm. Find the volume of the cylinder and its standard error. 6

(*c*) What is the physical significance of precision constant h ? Which one of the two sets of data having $h = 6$ and $h = 6.5$ respectively will have better precision ? 3

8. (*a*) Expland as a Fourier series 7,3

$$f(x) = x^2 + x, -\pi \le x \le \pi,$$

hence prove that :

$$1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots = \frac{\pi^2}{12}.$$

(*b*) Find Fourier cosine series of the function : 3,2

$$f(x) = \pi - x, 0 < x < \pi,$$

hence prove that :

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \dots\dots\dots = \frac{\pi^2}{8}.$$