This question paper contains 2 printed pages. **Your Roll No. ....**

Sl. No. Of Ques. Paper	:	**8381C**
Unique Paper Code	:	**222101**
Name of the Paper	:	**PHHT- 101 : Mathematical Physics - I**
Name of the Course	:	**B.Sc. (Hons) Physics Part I**
Semester	:	**I**
Duration	:	**3 hours**
Maximum Marks	:	**75**

Attempt five questions in all. Question No. 1 is compulsory.

Q 1.

a) Explain the physical significance of the curl of a vector field. **(3)**

b) Differentiate between systematic and random errors. **(3)**

c) A point has cartesian coordinates as (4, 3, 12). Write an expression for the position vector in cylindrical and spherical coordinates. **(3)**

d) Prove that $\sin x$ and $\cos x$ are orthogonal in the interval $(0, 2\pi)$. **(3)**

e) Evaluate the integral $\int_0^1 \left(\log \frac{1}{x}\right)^{n-1} dx, \quad n > 0$. **(3)**

Q 2.

a) Prove that a spherical coordinate system is orthogonal. **(4)**

b) Derive the expression for the divergence of a vector field in curvilinear coordinates and express it in spherical coordinates. **(4+3)**

c) Evaluate $\iiint_V \sqrt{(x^2 + y^2)}\,dxdydz$, where V is the region bounded by $z = x^2 + y^2$ and $z = 8 - x^2 - y^2$. **(4)**

Q 3.

a) Verify Green theorem in the plane for $\oint_c (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the region defined by $x = 0, y = 0, x + y = 1$. **(3+2)**

b) State and prove Stokes' theorem. **(2+5)**

c) Let $f = x^2yz - 4xyz^2$ be a scalar field. Find the directional derivative of f at P (1, 3, 1) in the direction of $2\hat{i} - \hat{j} - 2\hat{k}$. Also find the magnitude of maximum directional derivative. **(2+1)**

Q 4.

a) Prove the identity $\vec{\nabla}\cdot(\vec{A}\times\vec{B})=\vec{B}\cdot(\vec{\nabla}\times\vec{A})-\vec{A}\cdot(\vec{\nabla}\times\vec{B})$. **(5)**

b) Evaluate $\nabla^2\left[\vec{\nabla}\cdot\frac{\vec{r}}{r^2}\right]$, where $\vec{r}=x\hat{i}+y\vec{j}+z\vec{k}$. **(6)**

c) Assuming that $f(r)$ is differentiable, prove that $f(r)\vec{r}$ is irrotational. **(4)**

Q 5.

a) Find the Fourier series expansion of the output of a half wave rectifier. Draw the rectified wave function between 0 to T, where T is the time period. **(7+2)**

b) Obtain a cosine series expansion of the function $f(x)=1+x$ valid in the interval $0\le x\le 2$ and hence sum the series $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\cdots\cdots$. **(3+1)**

c) State the Dirichlet theorem so that a function can be expressed as a Fourier series. **(2)**

Q 6.

a) Prove the identity $\frac{\Gamma m\,\Gamma n}{\Gamma(m+n)}=B(m,n)$, where all the symbols have usual meaning. **(8)**

b) State the law of propagation of errors. The resistances of two resistors were determined several times giving the results $R_1=(3.52\pm0.01)\Omega$ and $R_2=(5.12\pm0.01)\Omega$. Calculate standard error in the total resistance R in series and in parallel. **(1+2+4)**

Q 7.

a) Prove that $\vec{\nabla}\left[\vec{M}\cdot\vec{\nabla}\left(\frac{1}{r}\right)\right]=\frac{3\left[\vec{M}\cdot\vec{r}\right]\vec{r}}{r^5}-\frac{\vec{M}}{r^3}$ **(6)**

b) Show that $B(m,n)=2\int_0^{\pi/2}(\sin\theta)^{2m-1}(\cos\theta)^{2n-1}\,d\theta$ **(4)**

c) Express the position and the velocity vectors of a particle in cylindrical coordinates. **(2+3)**