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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6201**

Unique Paper Code : **222101** **D**

Name of the Paper : **Mathematical Physics-I (PHHT-101)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

1. Do any *five* parts : 5×3=15

(*a*) Prove that :

$$\vec{\nabla} \times \left(\vec{\nabla} \phi \right) = 0.$$

(*b*) Define Jacobian. Calculate the Jacobian for a change from Cartesian (x, y) to polar coordinates (r, θ)

(*c*) What is the physical significance of precision constant 'h' ? Which *one* of the two sets of data having $h = 7$ and $h = 7.5$ respectively will have better precision ?

P.T.O.

(*d*) Prove that sin x and cos x are orthogonal in the interval $(0, 2\pi)$.

(*e*) State Dirichlet's conditions for Fourier series.

(*f*) Find the value of :

$$\beta\left(\frac{3}{2}, 2\right).$$

(*g*) Find the unit outward drawn normal to the surface :

$$(x - 1)^2 + y^2 + (z + 2)^2 = 9$$

at the point (3, 1, –4).

2. (*a*) Prove that :

$$\vec{\nabla} \cdot \left(\phi \vec{A}\right) = \left(\vec{\nabla} \phi\right) \cdot \vec{A} + \phi\left(\vec{\nabla} \cdot \vec{A}\right). \quad 5$$

(*b*) Show that :

$$\vec{A} = (x + 2y + 4z)\,\hat{i} + (2x - 3y - z)\,\hat{j} + (4x - y + 2z)\,\hat{k}$$

is irrotational. Find ϕ such that $\vec{A} = \vec{\nabla}\phi$. 5

(*c*) Evaluate :

$$\nabla^2(\ln r). \quad 5$$

3. (*a*) State and prove Gauss's Divergence theorem. 10

(*b*) If

$$\vec{F} = y\hat{i} + (x - 2xz)\hat{j} - xy\hat{k},$$

evaluate :

$$\iint_S \left(\vec{\nabla} \times \vec{F}\right) \cdot \hat{n}\, dS$$

where S is the surface of sphere $x^2 + y^2 + z^2 = a^2$ above the xy plane. 5

4. (*a*) Obtain an expression for the divergence of a vector in orthogonal curvilinear coordinates and express it in cylindrical coordinates. 8

(*b*) Evaluate using Green's theorem :

$$\int x^2 y\, dx + y^3\, dy$$

where C is rectangular curve formed by joining the points (0, 0), (1, 0), (1, 1) and (0, 1). 7

5. (*a*) Find the volume of the region common to the intersecting cylinders $x^2 + y^2 = a^2$ and $x^2 + z^2 = a^2$. 8

(*b*) Represent the vector :

$$\vec{A} = 2y\hat{i} - z\hat{j} + 3x\hat{k}$$

in cylindrical coordinates and determine A_ρ, A_θ and A_z. 7

6. (*a*) Expand in a Fourier series : 7

$$f(x) = \begin{cases} \sin x & 0 \le x \le \pi \\ -\sin x & -\pi \le x \le 0 \end{cases}$$

(*b*) Find the Fourier series for the periodic function *f*(*x*) defined by :

$$f(x) \begin{cases} = -\pi & -\pi < x < 0 \\ = x & 0 < x < \pi \end{cases}$$

and deduce :

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \text{--------} = \frac{\pi^2}{8}.$$ 8

7. (*a*) The radius of a wire is measured in cm as :

0.17, 0.15, 0.18, 0.19, 0.16, 0.17.

Find the mean radius and the standard error. 4

(*b*) Prove that :

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$ 6

(*c*) Evaluate :

$$\Gamma\left(\frac{1}{2}\right).$$ 5