This question paper contains **4** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **7876**

Unique Paper Code : **222101** **F-1**

Name of the Paper : **Mathematical Physics-I (PHHT-101)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **I**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all including Q. No. **1** which is compulsory.

1. Do any *five* of the following : 5×3=15

(*a*) Define a Jacobian. Calculate the Jacobian for a change from cartesian (x, y) to polar coordinates (r, θ) in two dimensions.

(*b*) What is Wronskian ? Calculate the Wronskian of x^n and $x^n \log x$.

(*c*) Find the minimum value of $u = x^4 + y^4 + z^4$ on the surface $xyz = c^3$, where c is a constant.

(*d*) Establish :

$$\delta(x^2 - a^2) = \frac{\delta(x - a) + \delta(x + a)}{2|a|}$$

(*e*) Show that the force :

$$\vec{F} = r^2 \vec{r}$$

is conservative.

P.T.O.

(*f*) Evaluate $\vec{\nabla}\left(\vec{F} \cdot \vec{r}\right)$, where $\vec{F}$ is a constant vector.

(*g*) Solve :

$$\cos(x + y)\, dy = dx$$

2. (*a*) Solve the following differential equations :

(*i*) $\sec^2 y \dfrac{dy}{dx} + 2x \tan y = x$

given that $y(1) = \dfrac{\pi}{4}$

(*ii*) $4\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + y = x(x + e^{-x/2})$ 5,5

(*b*) Solve the differential equation : 5

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x \log x$$

3. (*a*) Find the particular integral of the following differential equation : 5

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 5y = \frac{5}{4}e^{x/2} + 18\cos 4x - 71\sin 4x$$

(*b*) Solve : 5

$$x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = x^4$$

(*c*) Show that the following equation is exact and then solve it : 5

$$\{(x + 1)e^x - e^y\}\, dx = xe^y\, dy$$

4. (*a*) Evaluate the gradient ϕ, where ϕ is defined by :

$$\phi = \frac{\vec{p} \cdot \vec{r}}{r^3}$$

where $\vec{p}$ is a constant vector. 4

(*b*) Prove : 5

$$\frac{1}{2} \vec{\nabla} \times \left(\vec{\omega} \times \vec{r} \right) = \vec{\omega}$$

(*c*) Derive an expression for the divergence of $\vec{A}$ in spherical coordinates. 6

5. (*a*) Find the unit normal to the surface : 5

$$Z = \sqrt{\frac{3}{2}x^2 + \frac{3}{2}y^2}$$

at the point $\left(\sqrt{\frac{2}{3}}, 0, 1 \right)$.

(*b*) Which the following is not solenoidal ?

(*i*) $\dfrac{B_0 r_0^2 \, (x\hat{i} + y\hat{j} + z\hat{k})}{(x^2 + y^2 + z^2)^{3/2}}$

(*ii*) $B_0 \left(\dfrac{yz\hat{i}}{x^2 + y^2} - \dfrac{xz\hat{j}}{x^2 + y^2} \right)$

where B_0 and r_0 are constants. 4

(*c*) Obtain an expression for the curl of $\vec{A}$ in cylindrical coordinates. 6

6. (*a*) State Green's theorem in the plane. Verify Green's theorem in the plane for : 8

$$\oint_C [3x^2 - 8y^2]\, dx + [4y - 6xy] dy$$

where C is the boundary defined by :

$$x = 0,\ y = 0,\ x + y = 1.$$

(*b*) State the divergence theorem. Verify the divergence theorem for the vector field :

$$\vec{A} = 2x^2 y\hat{i} - y^2 \hat{j} + 4xz^2 \hat{k}$$

taken over the region in the first octant bounded by : 7

$$y^2 + z^2 = 9, x = 2.$$

7. (*a*) State Stokes' theorem. Verify Stokes' theorem for the vector field :

$$\vec{A} = (2x - y)\,\hat{i} - yz^2 \hat{j} - y^2 z\hat{k}$$

where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. 8

(*b*) Evaluate :

$$\iint_S \vec{A} \cdot \hat{n}\, dS \text{ where } \vec{A} = 18z\hat{i} - 12\hat{j} + 3y\hat{k}$$

and S is that part of the plane $2x + 3y + 6z = 12$ which is located in the first octant and $\hat{n}$ is the unit outward normal to S. 7