[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 1792 **GC-3** **Your Roll No................**

Unique Paper Code : 32221101

Name of the Paper : Mathematical Physics – I

Name of the Course : **B.Sc. (Hons.) Physics : Choice-based Credit System**

Semester : I

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **five** questions in all.

1. (a) By calculating the Wronskian of the functions e^x, xe^x and e^{-x}, check whether the functions are linearly dependent or independent. (4)

(b) Solve the inexact equation

$$(y^4 + 2y)dx + (xy^3 + 2y^4 - 4x)dy = 0 \quad (5)$$

(c) Solve the differential equation

$$\frac{d^2y}{dx^2} - y = e^x \cos x \quad (6)$$

2. (a) Solve the differential equation

$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} + \sin 3x \quad (8)$$

(b) Solve the differential equation using method of undetermined coefficients

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^2 + 2x + 4 \quad (7)$$

P.T.O.

3. (a) Solve the differential equation

$$\frac{d^2y}{dx^2}+3\frac{dy}{dx}=1-9x^2$$

given $y(0) = 0$ and $y'(0) = 1$. (8)

(b) Solve the differential equation using method of variation of parameters

$$\frac{d^2y}{dx^2}+a^2y = \text{cosec } ax$$ (7)

4. (a) Find

$$\frac{d}{dt}\left(\vec{V}.\frac{d\vec{V}}{dt}\times\frac{d^2\vec{V}}{dt^2}\right)$$

where $\vec{V}$ is a function of t.

(b) Find the Jacobian $J\left(\frac{x,y,z}{u,v,w}\right)$ of the transformation

$u = x^2 + y^2 + z^2$, $v = x^2 - y^2 - z^2$ and $w = x^2 + y^2 - z^2$.

(c) If $\vec{v} = \vec{w} \times \vec{r}$, find whether $\vec{v}$ is irrotational or not, where, $\vec{w}$ is a constant vector and $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(d) Find $\vec{\nabla} \times \left(f(r)\vec{r}\right)$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$.

(e) Find the directional derivative of a scalar function $\phi = 2xz - y^2$ at the point (1, 3, 2) in the direction of $xz\hat{i} + yz\hat{j} + xy\hat{k}$. (3×5=15)

5. (a) Prove that

$$\left(\vec{B}\times\vec{C}\right).\left(\vec{A}\times\vec{D}\right)+\left(\vec{C}\times\vec{A}\right).\left(\vec{B}\times\vec{D}\right)+\left(\vec{A}\times\vec{B}\right).\left(\vec{C}\times\vec{D}\right)=0$$ (5)

(b) Evaluate

$$\vec{\nabla}.\left[r\vec{\nabla}\left(\frac{1}{r^3}\right)\right]$$

where $r^2 = x^2 + y^2 + z^2$. (5)

(c) Evaluate

$$I = \oint_C \left(3x-8y^2\right)dx + \left(4y-6xy\right)dy$$

where C is the boundary of the region bounded by $x = 0$, $y = 0$ and $x + y = 1$. (5)

6. (a) Verify Stoke's theorem when

$$\vec{F} = \left(2xy - x^2\right)\hat{i} - \left(x^2 - y^2\right)\hat{j}$$

where C is the boundary of the region enclosed by $y^2 = x$ and $x^2 = y$. (10)

(b) Using Gauss Divergence theorem, prove that

$$\iiint_V \vec{\nabla}\times\vec{F}\,dV = \iint_S d\vec{S}\times\vec{F}$$

where V is the volume enclosed by surface S. (5)

7. (a) Derive an expression of curl of a vector field in orthogonal curvilinear coordinates. Express it in spherical coordinates. (6)

(b) Evaluate $\iiint_V \left(y^2 + z^2\right) dV$, where V is the volume bounded by the cylinder $x^2 + y^2 = a^2$ and the planes $z = 0$ and $z = h$. (6)

(c) Define the Dirac Delta function and establish

$$\int_{-\infty}^{+\infty} f(x)\delta'(x)\,dx = -f'(0)$$ (3)