This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **846**

Unique Paper Code : **222301** **G**

Name of the Paper : **PHHT-307 : Mathematical Physics III**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. 1 is compulsory.

Attempt *two* questions from each Section.

(The symbols have their usual meanings)

1. Attempt any *five* parts : 5×3

(*a*) Find the indicated roots of :

$$(-4 + 4i)^{\frac{1}{5}}$$

and locate them graphically.

(*b*) Graph the region represented by $1 < |z + i| < 2$.

(*c*) Show that :

$$P_n(-1) = (-1)^n.$$

P.T.O.

(d) Locate and name the singularities in the finite z-plane of :

$$\frac{\ln(z-3)}{(z^2+2z+2)^4}$$

(e) Discuss the nature of singularity for the function $ze^{\frac{1}{z}}$.

(f) Evaluate the following :

$$\lim_{z \to 2i} \frac{z^2+4}{2z^2+(3-4i)z-6i}$$

(g) Show that :

$$J_0'(x) = -J_1(x)$$

where symbols have their usual meanings.

Section A

2. (a) Derive Cauchy-Riemann conditions in Polar form. 4

(b) Given that $v = 3x^2y - y^3$ is a harmonic function, find u such that : 7

$$f(z) = u(x, y) + iv(x, y)$$

is analytic, hence write $f(z)$ in terms of z.

(c) Find the condition for which the function $f(z) = |z|^2$ is analytic. 4

3. Evaluate :

(a)

$$\frac{1}{2\pi i}\oint_C \frac{e^{zt}}{(z^2+1)^2}\,dz,\ t>0;$$

where C is the circle $|z| = 3$. 5

(*b*) If $f(z)$ be analytic inside and on the boundary of a simply connected region R, prove that :

$$f'(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{(z-a)^2}\, dz$$

where 'a' lies inside C. 6

(*c*) Find the value of the complex integral :

$$\oint_C \frac{dz}{z^2 - 2z}$$

where C is the circle $|z - 2| = 3$. 4

4. (*a*) Expand :

$$\ln \frac{(1+z)}{(1-z)}$$

in a Taylor series about $z = 0$. 7

(*b*) Find Laurent expansion of :

$$f(z) = \frac{1}{z(z-1)} \text{ valid for}$$

(*i*) $0 < |z| < 1$

(*ii*) $0 < |z - 1| < 1$ 8

5. Attempt any *two* parts using Contour Integration : 7½×2

(*a*) $\displaystyle\int_{-\infty}^{+\infty} \frac{dx}{(x^2 + 4x + 5)^2}$

(b) $$\int_0^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}$$

(c) $$\int_0^{\infty} \frac{\cos mx}{(x^2+1)} dx \qquad m > 0$$

Section B

6\. (a) Discuss the singularities of the equation : 3

$$x(x+1)y'' + (4x-1)y' + y = 0$$

(b) Solve the differential equation : 12

$$(3x)y'' + 2y' + y = 0$$

using Frobenius method.

7\. (a) Show that : 5

$$\frac{d}{dx}(x^n J_n(x)) = x^n J_{n-1}(x)$$

(b) Show that : 8

$$\int_{-\infty}^{\infty} e^{-x^2} H_n(x) H_m(x)\, dx = 2^n n! \sqrt{\pi}\, \delta_{mn}$$

where $H_n(x)$ and $H_m(x)$ are the Hermite Polynomials and

$$\delta_{mn} = 1 \text{ if } m = n$$
$$= 0 \text{ if } m \neq n$$

(c) Plot $J_0(x)$ as a function of x giving any two salient features. 2

8. (*a*) Starting from the generating function of $P_n(x)$, show that : 6

$$P_n(x) = \left(\frac{2n-1}{n}\right) x P_{n-1}(x) - \left(\frac{n-1}{n}\right) P_{n-2}(x).$$

(*b*) Express the function :

$$f(x) = 4x^3 + 6x^2 + 7x + 2$$

in a series of the form : 6 :75

$$\sum_0^\infty A_k P_k(x).$$

(*c*) Evaluate $H_3(x)$ using the expression for the generating function of Hermite Polynomials. 3

9. (*a*) Show that : 7

$$e^x \frac{d^n}{dx^n}(x^n e^{-x}) = L_n(x)$$

(*b*) Show that : 5×3

$$(1 - 2xt + t^2)^{-\frac{1}{2}} = \sum_0^\infty P_n(x) t^n$$

and hence derive expression for $P_2(x)$ and $P_3(x)$ where $P_n(x)$ are Legendre Functions. 8

P.T.O.