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Your Roll No......

1205

B.Sc. (Hons.) PHYSICS/II Sem. A

Paper—PHHT-203

MATHEMATICAL PHYSICS

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all including

Q. No. 1 which is compulsory.

1. Do any *five* parts : 3×5=15

(*a*) Differentiate between linear, non-linear and homogeneous differential equations with at least *one* example of each.

(*b*) Form the differential equation whose solution is :

$$y = ae^x \cos 3x + be^x \sin 3x,$$

where a and b are constants.

P.T.O.

(*c*) Solve the differential equation :

$$\left[y\left(1+\frac{1}{x}\right)+\cos y\right]dx + [x + \log x - x \sin y]dy = 0$$

(*d*) Solve the differential equation :

$y'' + 4y' + 29y = 0$, subject to initial condition $y(0) = 0$ and $y'(0) = 15$.

(*e*) Check whether the following functions are linearly dependent or independent :

$$e^{(p+iq)x},\ e^{(p-iq)x}.$$

(*f*) Find the extremal of $\int_{x_1}^{x_2} \sqrt{1+(y')^2}\,dx$, $y' = dy/dx$ and thus show that the shortest distance between two points in a plane is a straight line.

(*g*) Find the area of the largest rectangle that can be inscribed in a circle of radius r.

(*h*) Prove the following property of Poisson Bracket :

$$[uv, w] = [u, w]\,v + u[v, w].$$

2. Solve the following differential equations :

(*a*) $\frac{1}{1+y^2}\frac{dy}{dx} + 2x \tan^{-1} y = x^3$ 6

(*b*) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \cos x.$ 9

3. Solve the following differential equation :

(*a*) $x\frac{dy}{dx} - 2y = x^3 \cos 4x$ 5

(*b*) Using the method of undetermined coefficients, solve : 10

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = e^x + x.$$

4. Solve the following differential equations :

(*a*) $x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = x \log x$ 10

(*b*) $\frac{dy}{dx} + xy = x^3y^3.$ 5

5. (*a*) Solve the following differential equation using the method of variation of parameters : 10

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{2e^x}{x}.$$

(*b*) Solve the following differential equation : 5

$$\frac{d^2y}{dx^2} + 2y = x \sin x.$$

6. (*a*) Solve coupled differential equation : 10

$$\frac{dy}{dx} + y = z + e^x$$

$$\frac{dz}{dx} + z = y + 2e^x.$$

(*b*) Solve the differential equation : 5

$$x\frac{dy}{dx} + (1 - x)\, y = 3x^2y^2.$$

7. (*a*) Write the Euler-Lagrange's equation. Use it to determine the equation of the curve joining two given points which produce least area when it is revolved about the x-axis lying in the plane of the curve. 9

(*b*) Find the equation of motion of mass m, suspended at the end of a vertical spring of negligible mass using Lagrange's equation. 6

8. (*a*) Derive Hamilton's principle for a system of n-particles from Newton's laws of motion and also derive Lagrange's equation. 9

(*b*) Find the shortest distance from the origin to the curve : 6

$$5x^2 + 6xy + 5y^2 = 16.$$