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Your Roll No................

660

B.Sc. (Physical Science)/II Sem. **A**

Paper MAPT-202

MATHEMATICS-II (Calculus and Geometry)

(Admission of 2010 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

There are two Sections. Any *four* questions are to be attempted from Section-I. *All* questions are compulsory from Section-II. Marks are as indicated.

Section I

Attempt any *four* questions :

1. (*a*) (*i*) Use (ε, δ) definition to prove that :

$$\lim_{x \to +\infty} \frac{x}{x+1} = 1$$

(*ii*) Let the function be defines as :

$$f(x) = \begin{cases} 2x + 3, & x \le 4 \\ 7 + \dfrac{16}{x}, & x > 4 \end{cases}$$

Find the value of x (if any) at which f is not continuous. 2+2

Or

(*i*) Find δ such that :

$$\lim_{x \to 4} x^2 = 16; \ \varepsilon = 0.001$$

(*ii*) Show that the function defined by

$$f(x) = \begin{cases} x \sin \dfrac{1}{x}, & \text{when } x \ne 0 \\ 0, & \text{when } x = 0 \end{cases}$$

is continuous at $x = 0$. 2+2

(*b*) Find a value of k that will make the function continuous everywhere : 3

$$f(x) = \begin{cases} kx^2, & x \le 2 \\ 2x + k, & x > 2 \end{cases}$$

(*c*) Define Uniform Continuity. Show that the function $f(x) = 1/x$ is not unformly continuous on $]0, 1]$. 2+3½

2. (*a*) (*i*) Prove that 'A function which is derivable at a point is necessarily continuous at that point'. Is the converse true ? Justify.

(*ii*) Show $f(x) = |x| + |x-1|, \forall x \in \mathbb{R}$, is continuous but not derivable at $x = 0$. 3+2½

(*b*) State and prove Lagrange's mean value theorem and give its geometrical interpretation. 4

(*c*) Examine the validity of the hypothesis and the conclusion of Rolle's theorem for $f(x) = (x-a)^m (x-b)^n$, where m and n are positive integers, on $[a, b]$. 3

3. (*a*) Find the asymptotes of the curve :

$$x^3 + x^2y - xy^2 - y^3 + 2xy + 2y^2 - 3x + y = 0.$$ 2½

Or

$r = a \tan \theta$.

(b) (i) Determine the position and nature of the singular points on the curve :

$$x^2y^2 = (a + y)^2(b^2 - y^2).$$

(ii) Find the open intervals on which the function $y = 5 + 12x - x^3$ is concave up or concave down. Also find its points on inflexion. 3+3

(c) Sketch the graph of $r = \cos 2\theta$ in polar coordinates. 4

4. (a) Trace the curve $x(x^2 + y^2) = a(x^2 - y^2)$. 4

(b) Evaluate :

(i) $$\int \frac{dx}{(x^2 + 1)\sqrt{x^2 - 2}}$$

(ii) If $I_n = \int x^n (a - x)^{1/2} dx$, prove that :

$$(2x + 3)I_n = 2anI_{n-1} - 2x^n(a - x)^{3/2}.$$ 2+3

(c) Find the area included between the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ and its base. 3½

5. (*a*) Obtain reduction formula for $\int \sin^n x\,dx$, n being positive integer and hence evaluate $\int_0^{\pi/2} \sin^5 x\,dx$. 4

(*b*) Show that length of the loop of the curve $3ay^2 = x(x - a)^2$ is $\frac{4a}{\sqrt{3}}$. 4

(*c*) Find the volume of the solid generated by the revolution of the curve $(a - x)y^2 = a^2x$ about its asymptote. 4½

Section II

All questions are compulsory :

6. (*a*) Find the equation for the ellipse with foci $(0, \pm 2)$ and major axis with end points $(0, \pm 4)$.

Or

Determine $\nabla \cdot (\nabla \times F)$.

where $F(x, y, z) = \sin x\, i + \cos(x - y) j + zk$. 3½

(*b*) Sketch the graph of the equation :

$$y^2 - 8x - 6y - 23 = 0.$$ 6

(*c*) Explain reflection property of an ellipse together with a figure.

Or

For $F = F(x, y, z)$, $G = G(x; y, z)$ and $\varphi = \varphi(x, y, z)$, prove that $\text{div}\,(\varphi F) = \varphi\,\text{div}\,F + \nabla\varphi \cdot F$. 3

7. (*a*) Rotate the coordinate axes to remove the *xy*-term from the equation $31x^2 + 10\sqrt{3}xy + 21y^2 - 144 = 0$ and trace the conic.

Or

Find the equation of a sphere that has centre in the first octant and is tangent to each of the three coordinate planes. The distance from the origin to the sphere is $3 - \sqrt{3}$ units. 6

(*b*) (*i*) Sketch the parabola, determine focus, vertex, directrix

$(y-3)^2 = 6(x-2)$.

(*ii*) Show that for the radius vector $r = xi + yj + zk$.

$\nabla \|r\| = \frac{r}{\|r\|}$. 2½+1½

(*c*) Find an equation for a hyperbola with vertices (± 2, 0); foci (± 3, 0). 2½