This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1581**

Unique Paper Code : **222201** **C**

Name of the Paper : **PHHT-203 : Mathematical Physics II**

Name of the Course : **B.Sc. (Hons.) PHYSICS**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all including Q. No. **1** which is compulsory.

1. Do any *five* of the following : 5×3=15

(*i*) Determine the order, degree and linearity of the differential equation :

$$\left(\frac{d^3y}{dx^3}\right)^4 + \frac{d^2y}{dx^2} + y\frac{dy}{dx} = 0.$$

(*ii*) If y_1 and y_2 are the two solutions of a homogeneous differential equation :

$$y'' + p(x)y' + q(x)y = 0,$$

prove that $c_1y_1 + c_2y_2$ is also a solution of the equation. $\left(\text{here; } y' = \frac{dy}{dx}\right)$

(*iii*) Check whether the following functions are linearly independent or not :

$$e^x, \ x\,e^x, \ e^{-x}.$$

(iv) Find the maximum or minimum values of the function :

$$f(x, y) = y^2 + 4xy + 3x^2 + x^3.$$

(v) Check whether the following differential equation is exact, if so, solve it :

$$\left(e^x + \frac{2x}{y}\right) dx - \frac{x^2}{y^2} dy = 0.$$

(vi) Prove the following property of Poisson Bracket :

$$[uv, w] = [u, w]v + u[v, w].$$

(vii) Using Lagrange method of undetermined multipliers, find the maximum and minimum distance of the point (4, 6) from $x^2 + y^2 = 4$.

2. Solve the following differential equations :

(a) $\frac{dy}{dx} + xy = x^2 y^3$

(b) $\left(D^2 + 4\right) y = x \sin 2x.$ 5,10

3. (a) Solve :

$$\left(D^2 - 4D + 4\right) y = x^3 + e^x + \cos 2x.$$

(b) Solve :

$$\frac{d^2 y}{dx^2} + 4\frac{dy}{dx} + 4y = x^2,$$

where $y(0) = 0$ and $y'(0) = \frac{1}{2}$. 7,8

4. (*a*) Solve :

$$(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=2\sin\left[\log(1+x)\right].$$

(*b*) Using the method of undetermined coefficients, solve :

$$\frac{d^2y}{dx^2}=9x^2+2x-1.$$ 9,6

5. (*a*) Solve the following simultaneous equations :

$$\frac{dy}{dx}+y=z+e^x$$

$$\frac{dz}{dx}+z=y+2e^x.$$

(*b*) Using the method of variation of parameters, solve :

$$\frac{d^2y}{dx^2}-\frac{dy}{dx}-2y=4x^2.$$ 7,8

6. (*a*) Derive Euler-Lagrange equation for a function $f(x, y, y')$.

(*b*) Show that the path followed by a particle in vertical plane in sliding from one point to another in the absence of friction in the shortest time is a cycloid. 7,8

7. (*a*) A rectangular parallelopiped has a given surface area S. Determine its maximum volume using Lagrange Method of undetermined multiplier.

(*b*) If

$$u = \frac{a^3}{x^2} + \frac{b^3}{y^2} + \frac{c^3}{z^2},$$

where $x + y + z = 1$, then prove that the stationary value of u is given by :

$$x = \frac{a}{a+b+c},\ y = \frac{b}{a+b+c},\ z = \frac{c}{a+b+c}.$$

8. (*a*) Define Lagrangian brackets and prove that :

(*i*) $\{q_j, q_k\} = 0$

(*ii*) $\{p_j, p_k\} = 0$ and

(*iii*) $\{q_j, p_k\} = \delta_{jk}$.

(*b*) A block of mass m, connected at one end of the spring (spring constant k) is executing SHM (Fig. 1). Write the Lagrangian and hence determine the time period of oscillation. 7,8

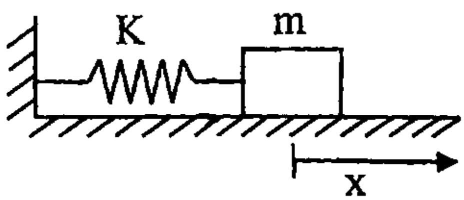

Fig. 1