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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **935**

Unique Paper Code : **222201** **E**

Name of the Paper : **PHHT-203 : Mathematical Physics II**

Name of the Course : **B.Sc. (Hons.) PHYSICS**

Semester : **II**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all. Question No. **1** is compulsory.

Attempt *four* questions from the rest of the question.

1. Do any *five* of the following : 5×3=15

(*a*) Determine the order, degree and linearity of the following differential equations :

$$\left(\frac{d^2x}{dy^2}\right)^2 + \frac{d^2x}{dy^2} + 4\frac{dx}{dy} = 0.$$

(*b*) What is Wronskian ? Calculate the Wronskian of the functions x, x^2, x^3.

(*c*) Two solutions of $y'' - 2y' + y = 0$ are e^x and xe^x. Is the general solution of the given equation $y = C_1 e^x + C_2 x e^x$? Discuss.

(*d*) Write the Euler-Lagrange's equation and explain the terms involved.

(*e*) Solve the differential equation :

$$(2xy + 1)dx + (x^2 + 4y)dy = 0.$$

(*f*) Find the shortest distance of the circle $x^2 + y^2 = 1$ from the point (2, 3).

(*g*) Define holonomic and non-holonomic constraints. Give *one* example each.

2. Solve the following differential equations :

(*a*) $(x^2 - ay)dx + (y^2 - ax)dy = 0$

(*b*) $\sec^2 y \, dy / dx + x \tan y = x^3$

(*c*) $\dfrac{dy}{dx} + y = \sin x;\ y(\pi) = 1.$ 5,5,5

3. Solve :

(*a*) $(D - 2)^2 y = 8(e^x + \sin 2x + x^2)$

(*b*) $100\dfrac{d^2N}{dt^2} - 20\dfrac{dN}{dt} + N = 0.$ 9,6

4. (*a*) Solve :

$$x^2 \frac{d^2y}{dx^2} + 5x\frac{dy}{dx} + 4y = x \log x.$$

(*b*) Using the method of undetermined coefficients, solve :

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = 4x + 10\sin x.$$ 9,6

5. (*a*) Solve the coupled differential equations :

$$\frac{dx}{dt} + \frac{dy}{dt} - 2y = 2\cos t - 7\sin t$$

$$\frac{dx}{dt} - \frac{dy}{dt} + 2x = 4\cos t - 3\sin t.$$

(*b*) Using the method of variation of parameters, solve :

$$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = e^{3x}.$$ 8,7

6. (*a*) Derive Euler-Lagrange's equation for a function $f(x, y, y')$.

(*b*) Show that the path followed by a particle in vertical plane in sliding from one point to another in the absence of friction in the shortest time is a cycloid. 7,8

7. (*a*) Using Lagrange's method of undetermined multipliers, find the largest product of x, y, z, when these are related as $x^2 + y^2 + z^2 = 9$.

(*b*) Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$ 7,8

P.T.O.

8. (*a*) Define Lagrangian Bracket and prove that :

(*i*) $\left[q_j, q_k\right] = 0$

(*ii*) $\left[p_j, p_k\right] = 0$

(*iii*) $\left[q_j, p_k\right] = \delta_{jk}$.

(*b*) A simple pendulum of mass *m* and length *l* executing SHM. Write the Lagrangian and hence determine the time period of small oscillation. 7,8