This question paper contains 4+1 printed pages]

Your Roll No................

1219

B.Sc. (Hons.)/II **A**

PHYSICS—Paper IX

(Mathematical Physics—II)

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Question No. 1 is compulsory.

Attempt *one* question from each Section.

1. Attempt any *five* parts : 5×2

(*a*) Find polar representation of $z = -2\sqrt{3} - 2i$ in terms of r and θ.

(*b*) Solve the equation :

$$z^3 + 2 = 0.$$

(*c*) Find the residue of the function :

$$f(z) = \frac{z^3}{z(z^2 + 4)}$$

at

$$z = 0 \text{ and } z = 2i.$$

(*d*) Express $J_2(x)$ in terms of $J_0(x)$ and $J_1(x)$.

(*e*) For Hermite polynomial H*n*(*n*) prove that :

$$H_n'(x) = 2nH_{n-1}(x)$$

(*f*) Determine the region in the *z*-plane represented by

$$4 < |z + i| < 8.$$

(*g*) Obtain all values of $\ln(\sqrt{3} - i)$ and find its principal values.

Section A

2\. (*a*) Prove that :

$$u = e^{-x}(x \sin y - y \cos y)$$

is harmonic and find v such that function $f(z) = u + iv$ is analytic. 5

(*b*) Show that, if $f(z)$ is analytic in a region R and on its boundary, then $\oint_C f(z)\, dz = 0$. 2

3. (*a*) If $f(z)$ is analytic in the entire z-plane and $z = a$ is a point in the z-plane, then prove that : 5

$$f'(a) = \frac{1}{2\pi i} \oint \frac{f(z)}{(z-a)^2}\, dz.$$

(*b*) Find the first four terms of Taylor series of the function : 2

$$f(z) = \ln(1 + z) \quad \text{around } z = 0.$$

Section B

4. Using contour integration evaluate any *two* of the following : 3½×2

(*a*) $\displaystyle\int_{-\infty}^{+\infty} \frac{\sin x}{x}\, dx$

(*b*) $\displaystyle\int_{0}^{\infty} \frac{dx}{(x^2 + 1)(x^2 + 4)^2}$

(*c*) $\displaystyle\int_{0}^{2\pi} \frac{d\theta}{3 - 2\cos\theta + \sin\theta}$

5. (*a*) What is meant by singular points of a second order differential equation. 2

(*b*) Solve the following differential equation by Frobenius power series method : 5

$$y'' + w^2 y = 0.$$

Section C

6. (*a*) Prove the recurrence relation : 3

$$x\,J_n'(x) = n\,J_n(x) - x\,J_{n+1}(x).$$

(*b*) Find the solution of : 4

$$y'' + \frac{1-2a}{x} y' + \left[(bc\,x^{c-1})^2 + \frac{(a^2 - p^2c^2)}{x^2} \right] y = 0.$$

7. (*a*) Prove that : 5

$$(n+1)L_{n+1}(x) = (2n+1-x)\,nL_n(x) - L_{n-1}(x).$$

(*b*) Prove that : 2

$$L_n(0) = 1.$$

Section D

8. Solve two dimensional wave equation for a circular membrane of radius 'a' oscillating symmetrically about origin, specifying the relevant boundary and initial conditions. 7

9. (*a*) Derive the heat conduction equation $\frac{\partial v}{\partial t} = h^2 \nabla^2 V$, where symbols have their usual meaning. 3

(*b*) Consider variable 1-dimensional linear heat flow in a rod with boundary conditions : Temperature = 0 at $x = 0$ and $x = s$ for all values of time and temperature = $F(x)$ for $t = 0$. Find the temperature of the rod at any time. 4