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Your Roll No.

1223

B.Sc. (Hons.) / II

A

PHYSICS – Paper XIII

(Mathematics - II)

Time : 3 Hours

Maximum Marks : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any two parts of each question.

1. (a) Define uniform convergence of a sequence of functions defined on an interval I. Show that the sequence $\{f_n\}$ where $f_n(x) = \frac{n}{n+x}$ is uniformly convergent in $[0, k]$, $k > 0$.

4 (1 + 3)

- (b) Show that the series $\sum_1^{\infty} \frac{x^n}{n(1+nx^2)}$ converges uniformly for all values of x .

4

- (c) Define radius of convergence of a power series. Let $\sum_{n=0}^{\infty} a_n x^n$ be a power series with non zero radius of convergence R . Prove that 4 (1 + 3)

$$\int_a^b \left[\sum_{n=0}^{\infty} a_n x^n \right] dx = \sum_{n=0}^{\infty} \frac{a_n}{n+1} (b^{n+1} - a^{n+1}).$$

where $-R < a < b < R$.

2. (a) Test the convergence of the following improper integrals :

(i) $\int_0^{\infty} e^{-x^2} dx$ (ii) $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^2}$ 4 (2 + 2)

- (b) Assuming the validity of differentiation under the sign of integral prove that 4 (3 + 1)

$$\int_0^{\pi/2} \frac{\text{Log}(1 + \cos \alpha \cos x)}{\cos x} dx = \frac{\pi^2 - 4\alpha^2}{8}$$

where $\alpha \in [0, \pi/2]$

Deduce that $\int_0^{\pi/2} \frac{\text{Log}(1 + \cos x)}{\cos x} = \frac{\pi^2}{8}$.

- (c) By using Beta Gamma functions show that

(i) $\int_0^1 \sqrt{1-x^4} dx = \frac{1}{12} \sqrt{\frac{2}{\pi}} \left[\frac{\Gamma(1/4)}{4} \right]^2$ 4 (3 + 1)

(ii) $\int_0^{\infty} e^{-x^2} dx = \frac{1}{2} \sqrt{\pi}$.

3. (a) State Baye's Theorem.
 A man is equally likely to choose any one of the routes A, B, C from his house to bus station, and his choice of route is not influenced by weather. If the weather is rainy the probabilities of missing the bus by routes A, B, C are $\frac{1}{20}$, $\frac{1}{10}$, $\frac{1}{5}$ respectively.
 He starts on a rainy day and misses the bus. What is the probability the route chosen was B? $3\frac{1}{2} (1 + 2\frac{1}{2})$
- (b) Two numbers are chosen at random, with replacement, from the first six positive integers. Compute the expected value of the absolute difference of these two numbers. Also find the variance. $3\frac{1}{2} (2 + 1\frac{1}{2})$
- (c) Find the moment generating function of a Binomial distribution with parameters n and p . Find the second central moment μ_2 . $3\frac{1}{2} (2\frac{1}{2} + 1)$
4. (a) The joint probability density function of two random variables x and y is given by
 $f(x, y) = 2 - x - y, 0 < x < 1, 0 < y < 1$
 $= 0, \text{ elsewhere}$
 Find the marginal distributions of x and y . Also find correlation coefficient between x and y . $4 (2 + 2)$
- (b) For 10 observations on price (x) and supply (y) the following data were obtained (in appropriate units):
 $\sum X = 130, \sum Y = 220, \sum X^2 = 2288,$
 $\sum Y^2 = 5506$ and $\sum XY = 3467$. Obtain the lines of regression. Estimate the supply when the price is 16 units. Also calculate correlation coefficient between them. $4 (2 + 1 + 1)$

- (c) Show that for normal distribution with mean μ and variance σ^2 the mean deviation about mean is approximately $\frac{4}{5}\sigma$. 4

5. (a) A random variable X has the probability density function $e^{-x}(0 \leq x < \infty)$. Use Chebyshev's inequality to obtain a lower bound to the probability of inequality $-1 \leq X \leq 3$ and compare it with actual value. ($3\frac{1}{2}(2\frac{1}{2} + 1)$)

- (b) 1000 apples kept under one type of storage were found to show 4% rotting. 1500 apples kept under another kind of storage showed 3% rotting. Can it be reasonably concluded that the second type of storage is superior to the first? 3 1/2

- (c) In a locality, 100 persons were randomly selected and asked about their educational achievements. The results are given as follows:

	Education		
	Middle	Metric	B.A.
Sex: Male	10	15	25
Female	25	10	15

Can you say that education depends on sex?

Given $\chi^2_{0.05} = 3.84$ for 1 d.f. $\chi^2_{0.05} = 5.99$

for 2 d.f. $\chi^2_{0.05} = 7.82$ for 3 d.f. 3 1/2