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Your Roll No. ....

5715

B.Sc. (Hons.) PHYSICS/III Sem. B

Paper PHHT-307

Mathematical Physics—III

(Admission of 2010 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Use separate answer-sheets for Section A and Section B.

Attempt *five* questions in all. Q. No. 1 is compulsory.

Attempt at least *one* question from each Section.

1. Attempt any *five* questions : 5×3

(*a*) Find all the roots of

$$\left(\frac{1+i}{1-i}\right)^{1/7}.$$

P.T.O.

(*b*) Show the region represented by $2 < |z - 4 - 5i| < 3$ in the complex plane.

(*c*) Solve the equation :

$$z^4 + z^2 + 1 = 0.$$

(*d*) Evaluate :

$$\frac{1}{2\pi i} \oint_C \frac{e^z}{z - 2i} \, dz$$

if C is given by :

(*i*) $|z| = 1$

(*ii*) $|z| = 3$.

(*e*) Prove the recurrence relation :

$$xJ'_n(x) = nJ_n(x) - xJ_{n+1}(x) .$$

(*f*) Prove that every polynomial equation of the form :

$$P(z) = a_0 + a_1 z + a_2 z^2 + \ldots\ldots\ldots + a_n z^n = 0$$

has exactly n roots, when $n \geq 1$ and $a_n \neq 0$.

(*g*) Solve the equation for a damped harmonic oscillator :

$$\ddot{x} - a\dot{x} + bx = 0.$$

(*h*) Locate and name the singularities of :

$$\frac{\sin z}{z} \text{ and } \frac{z}{e^{1/z} - 1}.$$

Section A

2. (*a*) Derive the necessary conditions in polar form for a complex function $f(z)$ to be analytic in a given region. 6

(*b*) If

$$f(z) = \frac{1}{(z - z_0)^m}$$

where m is an integer and z_0 is a constant. Prove that :

$$\oint_C f(z)\, dz = 0, \quad \text{for } m \neq 1$$

$$= 2\pi i, \quad \text{for } m = 1$$

where C is given by $|z - z_0| = \rho$, with the integral being evaluated clockwise. 4

(*c*) Show that :

$$u = 3x^2y + 2x^2 - y^3 - 2y^2$$

is harmonic and hence find its conjugate harmonic v. 5

3. (*a*) If a function $f(z)$ is analytic in a domain D, then show that it has derivatives of all orders in D, which are also analytic in D. Prove that $f'(z)$ at a point z_0 in D is given by :

$$f'(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)\,dz}{(z - z_0)^2}$$

when C lies in D. 9

(*b*) Evaluate :

$$\frac{1}{2\pi i} \oint_C \frac{\cos \pi z}{z^2 - 1} dz$$

when C is a rectangle with vertices at $2 \pm i$ and $-2 \pm i$. 6

4. (*a*) State and prove the Taylor's Theorem for complex analytic functions. 9

(*b*) Expand

$$f(z) = \frac{z}{z-3}$$

in a Laurent's series valid for :

(*i*) $|z| < 3$

(*ii*) $|z| > 3$

and locate the singularities. 6

5. Evaluate any *two* of the following using Contour integration : 2×7½

(*a*) $\int_0^\infty \frac{\cos mx}{x^2+1}\,dx$

(*b*) $\int_0^{2\pi} \frac{\sin^2\theta}{5-4\cos\theta}\,d\theta$

(c) $\int_0^\infty \frac{dx}{1 + x^4}$

(d) $\int_0^\infty \frac{\sin x}{x} \, dx.$

Section B

6. (a) Locate and name the singularities of the equation :

$$\left(1 - x^2\right)^2 y'' + x\left(1 - x\right) y' + \left(1 + x\right) y = 0. \qquad 3$$

(b) Using Frobenius method obtain the solution of the following equation about $x = 0$:

$$\left(x^2 - 1\right) x^2 y'' - \left(x^2 + 1\right) xy' + \left(x^2 + 1\right) y = 0. \qquad 12$$

7. (a) Prove that $J_n(x)$ is the coefficients of t^n in the expansion of :

$$\exp\left(\frac{1}{2} x \left(t - \frac{1}{t}\right)\right). \qquad 10$$

(b) Prove that :

$$nP_n(x) = xP_n'(x) - P_{n-1}'(x). \qquad 5$$