 Your Roll No. ....

Sl. No. Of Ques. Paper : **8401C**
Unique Paper Code : **222301**
Name of the Paper : **PHHT-307 :Mathematical Physics - III**
Name of the Course : **B.Sc. (Hons.) Physics Part II**
Semester : **III**
Duration : **3 hours**
Maximum Marks : **75**

Attempt five questions in all. Q.No. 1 is compulsory.

Attempt two questions from each Section.

Q.1. Do any five parts.

(a) Find the principal value of $\ell n(-1)$

(b) Prove that $|z_1 - z_2| \geq |z_1| - |z_2|$

(c) Locate and name the singularities in the finite z plane of

$$\frac{\ell n(z-2)}{(z^2+2z+2)^4}$$

(d) Describe graphically the region $1 < |z + i| \leq 2$

(e) Find

$$\mathcal{L}t_{z \to e^{\frac{i\pi}{6}}} \left[\frac{\left(z - e^{\frac{i\pi}{6}}\right).z}{(z^6+1)} \right]$$

(f) Show that $P_n(1) = 1$

(g) Show that $J_n(-x) = (-1)^n J_n(x)$

(3 x 5)

SECTION A

Q.2.

(a) Prove that the function $u = 2x(1-y)$ is harmonic. Find a function v such that $f(z) = u + iv$ is analytic.

$\left(7\frac{1}{2}\right)$

(b) Show that $f(z) = e^z$ is analytic and find its derivative by using first principle or writing $f(z)$ in terms of u and v.

$\left(7\frac{1}{2}\right)$

Q.3.

(a) If $f(z)$ is analytic inside and on the boundary C of a simply connected region R, prove that

$$f^n(z_0) = \frac{n!}{2\pi i}\oint_C \frac{f(z)dz}{(z-z_0)^{n+1}}; \quad n = 1, 2 \ldots\ldots$$

where z_0 lies inside C

$\left(7\frac{1}{2}\right)$

b) Evaluate

$$\oint_C \frac{e^{3z}dz}{(z-\pi i)}$$

using Cauchy's integral formula; if C is an ellipse $|z-2|+|z+2| = 6$

$\left(7\frac{1}{2}\right)$

Q.4

(a) If $f(z)$ is analytic inside and on the boundary of the ring shaped region R bounded by two concentric circles C_1 and C_2 with centre at 'a' and respective radii r_1 and r_2 ($r_1 > r_2$), then for all z in R, prove that

$$f(z) = \sum_{n=0}^{\infty} a_n(z-a)^n + \sum_{n=1}^{\infty} \frac{a_{-n}}{(z-a)^n}$$

where symbols have their usual meanings.

(10)

(b) Expand $f(z) = \sin z$ in a Taylor Series about $z = \pi/4$.

(5)

Q.5.

Do any **two** parts using Contour Integration:

(a). $$\int_0^\infty \frac{dx}{(x^4+x^2+1)}$$

(b). $$\int_0^{2\pi} \frac{\cos 3\theta \ d\theta}{(5 - 3\cos\theta)}$$

(c). $$\int_0^{\infty} \frac{Sinx}{x} dx$$

$\left(7\frac{1}{2} \times 2\right)$

SECTION B

Q.6.

Locate and name the singularities of the equation

$$x(x-1)y'' + (3x-1)y' + y = 0$$

and find its general solution using Frobenious method.

(5, 10)

Q.7.

(a) Prove the Rodrigue's formula for Hermite Polynomial $H_n(x)$,

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$$

$\left(7\frac{1}{2}\right)$

(b) Derive the recurrence relation for Legendre Polynomial

$$(1 - x^2)P_n'(x) = nP_{n-1}(x) - nxP_n(x)$$

$\left(7\frac{1}{2}\right)$

Q.8.

(a) Prove that

$$\int_{-\infty}^{\infty} e^{-x^2} H_m(x) H_n(x) dx = \sqrt{\pi}\, 2^n n!\, \delta_{mn}$$

where $H_m(x)$ and $H_n(x)$ are the Hermite Polynomial and $\delta_{mn} = 1 \quad if\ m = n$
$= 0 \quad if\ m \neq n.$

$\left(7\frac{1}{2}\right)$

(b) Show that

$$J_{1/2}^2(x) + J_{-1/2}^2(x) = \frac{2}{\pi x}$$

where $J_{1/2}(x)$ and $J_{-1/2}(x)$ are Bessel's functions

$\left(7\frac{1}{2}\right)$