[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8404 **C** **Roll No............**

Unique Paper Code : 235362

Name of the Paper : PHHT-310: Mathematics – I

Name of the Course : B.Sc. (Hons.) Physics, Part II

Semester : III

Duration : 3 Hours

Maximum Marks : 75

<u>Instructions for Candidates</u>

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. **All** questions are compulsory.

1. Attempt any **two** parts :

(a) Prove that the limit of a sequence if it exists is unique. Using the definition of the convergence of a sequence prove that

$$\lim_{n\to\infty} \frac{(-1)^n}{n} = 0$$ (7½)

(b) Show that the sequence $<a_n>$ defined by $a_{n+1} = \sqrt{7 + a_n}$, $a_1 = \sqrt{7}$ converges to the positive roots of the equation $x^2 - x - 7 = 0$. (7½)

(c) Prove that $\lim_{n\to\infty} n^{\frac{1}{n}} = 1$. Use this result to prove that

$$\lim_{n\to\infty} \frac{1}{n}\left[1 + 2^{\frac{1}{n}} + \cdots + n^{\frac{1}{n}}\right] = 1$$ (7½)

2. Attempt any **two** parts :

(a) Prove that a necessary and sufficient condition for the series $\sum_{n=1}^{\infty} u_n$ to be convergent is that given $\in > 0 \ \exists \ m \in \boldsymbol{N}$ such that

$|u_{m+1} + u_{m+2} + \cdots + u_n| < \in \quad \forall \ n \geq m.$ (7)

(b) Test the convergence of any **two** :

(i) $\sum_{n=1}^{\infty} \left(1 + \frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$

(ii) $\frac{x}{1} + \frac{1}{2} \cdot \frac{x^3}{3} + \frac{1.3}{2.4}\frac{x^5}{5} + \frac{1.3.5}{2.4.6}\frac{x^7}{7} + \cdots$

(iii) $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n^2}{(n+1)!}$ (3½,3½)

(c) Show that the series $x - \frac{x^3}{3} + \frac{x^5}{5} - \cdots$ converges if $-1 \leq x \leq 1$. (7)

3. Attempt any **three** parts :

(a) Show that a function continuous on a closed and bounded interval [a,b] is uniformly continuous on [a,b]. (6)

(b) Prove that sin x lies between $x - \frac{x^3}{6}$ and $x - \frac{x^3}{6} + \frac{x^5}{120}$ $\forall \ x \in \boldsymbol{R}$. (6)

(c) State Intermediate Value theorem. Let f be a continuous function on [0,1] and let $f(x)$ be in [0,1] for each x in [0,1]. Prove that $f(x) = x$ for some x in [0,1]. (6)

(d) Examine the function f where

$$f(x) = \begin{cases} x\left(\dfrac{e^{\frac{1}{x}} - e^{-\frac{1}{x}}}{e^{\frac{1}{x}} + e^{-\frac{1}{x}}}\right), & x \neq 0 \\ 0, & x = 0 \end{cases}$$

as regards continuity and derivability at the origin. (6)

4. Attempt any **two** parts :

(a) Show that for the function $f(x, y) = \begin{cases} \dfrac{xy}{\sqrt{x^2 + y^2}}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$

f_x *and* f_y exist at (0,0) but f is not differentiable at (0,0). (5)

(b) Expand $xy^2 + 2x - 3$ in powers of $x - 2$ and $y + 1$. (5)

(c) For the function $f(x, y) = 2y^2x - yx^2 + 4xy$, locate all relative(local) maxima, relative (local) minima and saddle points if any. (5)

5. Attempt any **three** parts :

(a) Prove that every continuous function defined on an interval [a,b] is Riemann integrable in [a,b]. (6)

(b) If a function f is defined in the interval [0,1] by the condition that

$f(x) = (-1)^{r-1}$ when $\dfrac{1}{r+1} < x < \dfrac{1}{r}$ where r is a positive integer,

then prove that f is Riemann integrable in [0,1] and $\int_0^1 f(x)dx = log4 - 1$.

(6)

(c) Find upper and lower Darboux integrals for $f(x) = x^2$ in $[0, b]$. (6)

(d) Suppose f and g are continuous functions on [a,b] such that $\int_a^b f\,dx = \int_a^b g\,dx$.

Prove that there exists x in [a,b] such that $f(x) = g(x)$. (6)