This question paper contains 4 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6207**

Unique Paper Code : **222301** **D**

Name of the Paper : **Mathematical Physics-III : PHHT-307**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all. Question No. **1** is compulsory.

Attempt *two* questions from each Section.

1. Do any *five* : 5×3

(*a*) Find the polar form of complex number $z = -\sqrt{3} - i$.

(*b*) Determine the principal value of $(1 + i)^i$.

(*c*) Check whether the function $u = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic or not.

(*d*) Find the value of

$$\oint_C \frac{\sin^6 z}{z - \frac{\pi}{6}} dz$$

if C : $|z| = 1$.

(*e*) Find the residue of

$$f(z) = \frac{\sin z}{z^2}$$

at the pole $z = 0$.

(*f*) Prove that :

$$P_n'(-1) = (-1)^{n+1} \frac{n(n+1)}{2}.$$

P.T.O.

(*g*) Using generating function, find $H_2(x)$.

(*h*) Find $J_{-\frac{1}{2}}(x)$.

Section A

2. (*a*) Find all the values of $\sin^{-1} 2$. 7

(*b*) Verify whether the function $f(z) = e^{z^2}$ is analytic or not. 8

3. (*a*) If $f(z)$ is analytic inside and on a simple closed curve C, then prove that :

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z - a} \, dz.$$ 7

(*b*) If $t > 0$, prove that :

$$\frac{1}{2\pi i} \oint_C \frac{z e^{zt}}{(z+1)^3} \, dz = \left(t - \frac{t^2}{2}\right) e^{-t},$$

where C : $|z| = 2$. 4

(*c*) Locate and name all the singularities of the function

$$f(z) = \frac{z}{e^z - 1}$$

in the finite z-plane. 4

4. (*a*) Using Taylor's Theorem, find first three non-zero terms of $f(z) = \sec z$, about $z = 0$. 7

(*b*) Expand

$$f(z) = \frac{z}{(z-1)(2-z)}$$

in the Laurent series valid for :

(*i*) $|z| < 1$

(*ii*) $|z - 1| > 1$. 4,4

5. Evaluate any *two* using Contour Integration :

(*a*) $$\int_0^\infty \frac{dx}{\left(x^2+4\right)^2}$$

(*b*) $$\int_0^{2\pi} \frac{dx}{5+4\cos x}$$

(*c*) $$\int_0^\infty \frac{x\sin 2x}{x^2+9}\,dx.$$ 7.5,7.5

Section B

6. (*a*) Discuss whether each of the points +1, 0, –1 is ordinary, regular singular point or irregular singular point of the differential equation :

$$x^2\left(1-x^2\right)y'' + \frac{2y'}{x} + 4y = 0.$$ 3

(*b*) Using Frobenius method, solve the differential equation :

$$xy'' + y' - xy = 0.$$ 12

7. (*a*) Using generating function of Bessel's function, show that :

$$\sin(x\sin\theta) = 2\left[J_1(x)\sin\theta + J_3(x)\sin 3\theta + J_5(x)\sin 5\theta + \ldots\ldots\ldots\right]$$

and hence prove that :

$$x\cos x = 2\left[1^2 J_1(x) - 3^2 J_3(x) + 5^2 J_5(x) - \ldots\ldots\ldots\right].$$ 5,2

(*b*) Derive the following recurrence relation :

$$2J_n'(x) = J_{n-1}(x) - J_{n+1}(x).$$ 4

(*c*) Using generating function prove that :

$$J_n(x+y) = \sum_{p=-\infty}^{\infty} J_p(x)\, J_{n-p}(y).$$ 4

8. (*a*) If

$$f(x) = \sum_{n=0}^{\infty} a_n P_n(x), \ -1 < x < 1,$$

then show that :

$$a_n = \frac{2n+1}{2} \int_{-1}^{1} f(x)\, P_n(x)\, dx.$$

Hence, expand $f(x) = x^2 + 5x - 2$ in series of the form $\sum_{n=0}^{\infty} a_n P_n(x)$. 3,4

[Given : $P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = (3x^2 - 1)/2$.]

(*b*) Prove that the orthogonality condition for Legendre's polynomial is given by :

$$\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = \frac{2}{2n+1} \delta_{m,n}.$$ 8

9. (*a*) Prove that $\frac{H_n(x)}{n!}$ is the coefficient of t^n in the expansion of $e^{2xt - t^2}$ in ascending powers of t. 7

(*b*) Show that the Rodrigue's formula for Laguerre's polynomial is given by :

$$L_n(x) = \frac{e^x}{n!} \frac{d^n}{dx^n}\left(x^n e^{-x}\right).$$ 8