2461

Unique paper Code:

Name of the Paper: **MATHEMATICAL PHYSICS - III**

Name of the Course: **B.Sc (Hons.) Physics IIIrd Year**

Semester: **Part - III**

Duration: **3 Hours**

Maximum Marks : **38**

E

Attempt five questions in all.

Question No. 1 is compulsory

Attempt one question from each section.

Q.1 Attempt any five of the following: (2x5)

a) If H is a Hermitian matrix, prove that e^{iH} is unitary.

b) Check whether {(1,1,2), (2,2,1), (1,2,2)} forms a basis for R^3.

c) Prove that $x\delta(x) = 0$.

d) If $L(F(t)) = f(s)$ then show that $\left[\int_0^t F(u)du\right] = \frac{f(s)}{s}$.

e) Prove that kronecker delta is a symmetric isotropic tensor of order 2.

f) Show that similar matrices have the same eigenvalues.

g) Show that the Fourier transform of a Gaussian Function is itself a Gaussian function.

h) Show that the set of cube roots of unity viz $S = \{1, w, w^2\}$ forms a group under multiplication.

Section A

Q.2 (a) (a) Find the dimension and basis of the solution space V of the system of equations: (4)

$$x_1 + 2x_2 - 4x_3 + 3x_4 - x_5 = 0$$

$$x_1 + 2x_2 - 2x_3 + 2x_4 + x_5 = 0$$

$$2x_1 + 4x_2 - 2x_3 + 3x_4 + 4x_5 = 0$$

(b) Determine whether the vectors u = (1,1,2), v = (2,3,1) and w = (4,5,5) in R^3 are linearly dependent. (3)

Q.3 (a) A transformation T is defined by (4)

$$(x,y)T = \frac{1}{\sqrt{2}}\,(x-y, x+y)$$

(i) Show that T is linear

(ii) Find the matrix M represented by T w.r.t. the bases (1,0) and (0,1)

(b) If K is skew Hermitian and (K - I) is non-singular, show that $U = (K + I)(K - I)^{-1}$ is unitary.

(3)

Section B

Q.4 (a) If a matrix A is given by (5)

$$A = \frac{1}{2}\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$$

Fine e^A and 4^A.

(b) Prove that if A is an orthogonal matrix, then A^{-1} is also orthogonal. (2)

Q.5 (a) Determine the eigenvalues and corresponding eigenvectors of the matrix (5)

$$A = \begin{bmatrix} a & b & b \\ b & a & b \\ b & b & a \end{bmatrix}$$

Where a and b are scalars and $b \neq 0$.

(b) Show that $A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is not diagonalizable. (2)

Section C

Q.6 (a) Define moment of inertia tensor and show that it is a symmetric tensor of order 2. (5)

(b) If T_{ij} is a skew-symmteric tensor of order 2, prove that (2)

$$(\delta_{ij}\delta_{lk} + \delta_{il}\delta_{jk})\, T_{ik} = 0$$

Q.7 (a) Show that , (4)

$$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a}.\vec{c})\vec{b} - (\vec{b}.\vec{a})\vec{c}$$

(b) Show that if $A_{ijkl......}$ and $B_{pqrs......}$ are two tensors of order m and n respectively, then their product $C_{ijkl......pqrs} = A_{ijkl......} B_{pqrs......}$ is a tensor of order m + n. (3)

Section D

Q.8 (a) Find the Fourier cosine transform of $f(t) = e^{-pt}, p > 0$ (5)

Hence evaluate the integral $\int_0^{\infty} \frac{\cos(wt)}{p^2+w^2}\, dw$

(b) Prove that : $\int_{-\infty}^{+\infty} f'(x) \cdot \delta(x) dx = -f'(0)$ (2)

Q.9(a) Solve $y''(t) + y(t) = t,\ y(0) = 1, y'(0) = -2$ using Laplace Transforms. (4)

(b) Given that f(t) is a periodic function with period T,find its Laplace Transform. (3)