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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1465**

Unique Paper Code : **2221301** **F-7**

Name of the Paper : **Mathematical Physics II (439)**

Name of the Course : **B.Sc. (H) Physics Admitted Previously under FYUP**

Semester : **III**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

1. Attempt any *five* questions : 3×5=15

(*a*) Show that an even function can have no sine terms in its Fourier expansion.

(*b*) Write Fourier sine series of function $f(t)$ having period 2T and find its Fourier coefficient.

(*c*) Write Legendre's Equation and show that :

$$P_n'(1) = \frac{1}{2} n (n+1).$$

(*d*) Show that :

$$J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x.$$

(*e*) Reduce the following differential equations to Bessel equation :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + (kx^2 - n^2)y = 0.$$

(*f*) Evaluate :

$$\int_0^1 (x \ln x)^3 \, dx.$$

(*g*) Write three-dimensional Laplace's equation in spherical coordinates. Mention a physical problem involving such Laplace's equation.

2. An alternating current after passing through a rectifier has the form :

$$i(\theta) = \begin{cases} I\sin\theta & 0 < \theta < \pi \\ 0 & \pi < \theta < 2\pi \end{cases}$$

(*a*) Sketch its graph from -4π to 4π. 2

(*b*) State whether this function is odd, even or neither odd nor even. 1

(*c*) Find the Fourier series of the function. 12

3. (*a*) Obtain Parseval's Formula : 5

$$\int_{-l}^{l} [F(x)]^2 \, dx = l\left\{\frac{1}{2}a_0^2 + \sum_{n=1}^{\infty}\left(a_n^2 + b_n^2\right)\right\}$$

Assuming the Fourier series corresponding F(x) converges uniformly to F(x) in $(-l, l)$ and the integral also exists.

(*b*) Obtain Fourier sine series for $f(x) = 1$ in $0 < x < \pi$ and deduce that : 10

$$\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots\ldots$$

4. (*a*) Discuss the regular and irregular singular points of the following differential equation : 3

$$(1 + x^2)x^2 \frac{d^2y}{dx^2} + \frac{5dy}{dx} - y = 0.$$

(*b*) Using Frobenius method solve in series the Laguerre's differential equation : 12

$$x\frac{d^2y}{dx^2} + (1 - x)\frac{dy}{dx} + \lambda y = 0$$

Here λ is a constant.

5. (*a*) Prove that : 5

$$(n + 1)p_{n+1}(x) = (2n + 1)xP_n(x) - nP_{n-1}(x).$$

(*b*) Using (*a*) find $P_3(x)$, where $P_0(x) = 1$ and $P_1(x) = x$. 4

(*c*) Prove that : 6

$$\int_{-1}^{1} P_m(x)P_n(x)\,dx = 0 \quad \text{if } m \neq n.$$

6. (*a*) Verify that : 5

$$e^{\frac{x}{2}(t-\frac{1}{t})} = \sum_{n=-\infty}^{\infty} J_n(x)t^n.$$

(*b*) Using (*a*) show that : 2

$$J_n(0) = 0 \quad \text{for } n = 1, 2, 3, \ldots\ldots$$

(*c*) Given : 3

$$\int_0^\infty \frac{x^{p-1}}{1+x}\,dx = \frac{\pi}{\sin p\pi} \qquad \text{where } 0 < p < 1.$$

Show that :

$$\Gamma(p)\Gamma(1-p) = \frac{\pi}{\sin p\pi}.$$

(*d*) Evaluate : 5

$$\int_0^\infty \frac{x\,dx}{1+x^6}.$$

7. A string is stretched and fastened to two points '*l*' apart. Motion is started by displacing the string in the form :

$$u = a\sin\frac{\pi x}{l}$$

from which it is released at time $t = 0$. Find the displacement $u(x, t)$ of any point at distance '*x*' from one end and at any time '*t*'. 15