

[This question paper contains 2 printed pages.]

Sl. No. : 2070 GC-3 Your Roll No.....
Unique Paper Code : 32221301
Name of Paper : Mathematical Physics-II
Name of the Course : B.Sc (Hons) Physics
Scheme of Examination : CBCS Part-III (Sem-III)
Duration : 3 hours
Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No at the top of your Question Paper.
2. Question no 1 is compulsory. Attempt five questions in all.
3. In the question paper $y \equiv y(x), y' \equiv \frac{dy}{dx}$ and $y'' \equiv \frac{d^2y}{dx^2}$.

1. Attempt any five questions

- a) Find the indicial equation and its roots for the given differential equation

$$x^2 y'' + 3xy' + 4xy = 0$$

- b) Evaluate the value of $\frac{\Gamma(3)\Gamma(3/2)}{\Gamma(9/2)}$.

- c) Find the value of $P_n(-1)$.

- d) State the Dirichlet's conditions.

- e) Express $f(x)$ as a linear combination of Legendre Polynomials where

$$f(x) = 5x^3 + 2x^2 - 7x + 4$$

- f) Given that $f(x) = f(x+2\pi)$ and if

$$f(x) = |x| \text{ if } -\pi < x < \pi$$

Graphically represent the function in the interval $(-5\pi, 5\pi)$

- g) Evaluate $\int_0^2 x^3 \sqrt{8-x^3} dx$

5x3=15

2. A rectangular homogeneous membrane of length a and breadth b is fixed at the boundaries. The membrane is free to oscillate vertically with respect to its plane. Initially, the membrane was at rest and its initial position is given as $f(x,y)$. Find the displacement at any time 't' and at position (x,y).

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P.T.O.

3. Find the Fourier series expansion of the function

$$f(x) = \begin{cases} -kif & -\pi < x < 0 \\ kif & 0 < x < \pi \end{cases} \text{ and } f(x + 2\pi) = f(x)$$

Hence find the value of $\sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$ 12+3=15

4. Prove that

$$\int_{-1}^1 P_m(x)P_n(x)dx = \frac{2}{2n+1} \delta_{mn} \quad 12 \times 7.5 = 15$$

Q5.a) Solve the given differential equation

$$y'' - 2xy' + 2\lambda y = 0$$

λ is a real constant. For $\lambda = n = 0, 1, 2, 3, \dots$, show that one of the solution is a polynomial of order n . Name the polynomial.

b) Show that

$$J_{1/2}(x) = \sqrt{\frac{2}{\pi x}} \sin(x) \quad 12+3=15$$

6. a) For Legendre polynomial $P_n(x)$, prove that

$$nP_n = (2n-1)xP_{n-1} - (n-1)P_{n-2}$$

b) Show that $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

c) Show that $\frac{d}{dx}\{x^n J_n(x)\} = x^n J_{n-1}(x)$ 3x5=15

7. Solve the differential equation 15

$$x^2 y'' + xy' + (x^2 - 4)y = 0$$