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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : 849

Unique Paper Code : 235362 **G**

Name of the Paper : Mathematics–I [PHHT-310]

Name of the Course : B.Sc. (H) Physics

Semester : III

Duration : 3 Hours Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any *two* parts from each question.

1. (*a*) Define the convergence of a sequence. Prove that : 7½

$$\lim_{n \to \infty} (a)^{1/n} = 1, a > 0.$$

(*b*) Prove that every convergent sequence is a Cauchy sequence. Apply it to prove that the sequence $\langle a_n \rangle$ defined by : 7½

$$a_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n}$$

does not converge.

(*c*) Show that the sequence $\langle a_n \rangle$ defined by : 7½

$$a_{n+1} = \sqrt{7 + a_n}, \quad a_1 = \sqrt{7}$$

converges to a positive root of the equation $x^2 - x - 7 = 0$.

2. (*a*) Show that the series : 7½

$$1 + r + r^2 + r^3 + \dots + r^n \text{ .....}, (r > 0)$$

converges if $r < 1$ and diverges if $r \geq 1$.

(*b*) State Raabe's test. Show that the series : 7½

$$\sum \frac{3.6.9 \text{ .....} 3n}{7.10.13 \text{ ....}(3n + 4)} x^n, x > 0$$

converges for $x \leq 1$ and diverges for $x > 1$.

(*c*) State Leibnitz test for alternating series. Show that the series : 7½

$$\sum \frac{(-1)^{n+1}}{n^p}$$

is absolutely convergent for $p > 1$, but conditionally convergent for $0 < p \leq 1$

3. (*a*) Show that the function :

$$f(x) = \frac{1}{x^2}$$

is uniformly continuous on A = $[1, \infty)$, but it is not uniformly continuous on B = $(0, \infty)$ 7½

(*b*) Let R be the set of real numbers. State intermediate value theorem for a continuous function. Let the function $f : [-1, 1] \to$ R be defined as follows :

$$f(x) = \begin{cases} -1 - x, & -1 \leq x < 0 \\ 1 - x, & 0 \leq x \leq 1 \end{cases}$$

Show that f is not continuous at $x = 0$ and conclusion of intermediate value theorem does not hold for this function. 7½

(c) Obtain and plot the Taylor's polynomial of order 0, 1 and 2 generated by $f(x) = e^x$ at $x = 0$. Also obtain the Maclaurin series for this function. 7½

4. (a) Investigate the maxima and minima of the function : 7½

$$f(x, y) = x^3 + y^3 - 3x - 12y + 20$$

(b) Show that the function :

$$f(x,y) = \begin{cases} \dfrac{x^2 y}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0, & \text{if } (x, y) = (0, 0) \end{cases}$$

is continuous but not differentiable at origin. 7½

(c) State Young's theorem. For the following function :

$$f(x,y) = \begin{cases} \dfrac{(x^2 y + xy^2) \sin (x - y)}{x^2 + y^2}, & \text{if } (x, y) \neq (0, 0) \\ 0 & , \text{if } (x, y) = (0, 0) \end{cases}$$

prove or disprove : 7½

$$f_{xy}(0, 0) = f_{yx}(0, 0).$$

5. (a) If :

$$f(x) = \begin{cases} 1, & \text{when } x \text{ is rational} \\ -1, & \text{when } x \text{ is irrational} \end{cases}$$

then show that f is not Riemann integrable on any interval $[a, b]$. 7½ (*b*) If M, *m* are the bounds of an integrable function *f* on [*a*, *b*], then show that : 7½

$$m(b-a) \le \int_a^b f(x)\, dx \le M(b-a).$$

(*c*) If a function *f* is monotonic on [*a*, *b*], then prove that it is Riemann integrable on [*a*, *b*]. 7½