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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **1589**

Unique Paper Code : **222401** **C**

Name of the Paper : **Mathematical Physics IV (PHHT-411)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all taking at least *one* question from each Section.

Section A

1. (*a*) Let $V = \mathbf{R}^3$. Determine whether or not W is a subspace of V, where :

$$W = \{(a, b, c) : ab = 0\}.$$

(*b*) Let V be the set of all polynomials of degree ≥ 3. Determine if V(**R**) is a vector space.

(*c*) Find a basis and the dimension of the solution space W of homogeneous system : 5,5,5

$$x + 2y - z + 3s - 4t = 0$$

$$2x + 4y - 2z - s + 5t = 0$$

$$2x + 4y - 2z + 4s - 2t = 0.$$

2. (*a*) Consider $T : \mathbf{R}^3 \to \mathbf{R}^3$ defined by $T(x, y, z) = (x + y, 2z, 0)$. Determine whether or not T is a linear transformation.

(*b*) Find the matrix representation of the operator $T : \mathbf{R}^2 \to \mathbf{R}^2$ defined by :

$$T(x, y) = (3x - 4y, x + 5y) \text{ relative to}$$

(*i*) usual basis : $e = \{e_1 = (1, 0), e_2 = (0, 1)\}$

(*ii*) f-basis : $f = \{f_1 = (1, 3), f_2 = (2, 5)\}$. 5,10

Section B

3. (*a*) Show that matrix $A = \begin{bmatrix} \sqrt{2}/2 & -i\sqrt{2}/2 & 0 \\ i\sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ is a Unitary matrix.

(*b*) If H is a Hermitian matrix and U is a Unitary matrix, prove that $U^{-1}HU$ is Hermitian.

(*c*) Suppose λ is an eigen value of an invertible operator T. Show that λ^{-1} is an eigen value of T^{-1}. 5,5,5

4. (*a*) Determine the eigen values and eigen vectors of the matrix :

$$A = \begin{bmatrix} 0 & 2 \\ 3 & -1 \end{bmatrix}$$

Can matrix A be diagonalized ? If yes, find a diagonalizing matrix P and verify that P diagonalizes the given matrix A.

(*b*) An eigen value of a skew-Hermitian matrix is either zero or purely imaginary. 10,5

5. (*a*) Solve the following systems of differential equations using matrix method :

$$y_1' = -y_1 + 4y_2$$

$$y_2' = 3y_1 - 2y_2$$

subject to the initial conditions $y_1(0) = 3$ and $y_2(0) = 4$.

(*b*) Verify Cayley-Hamilton theorem for matrix :

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$$

and hence find A^{-1}. 10,5

Section C

6. Consider a uniform flexible chain hanging from a support under the action of gravity. At time $t = 0$, the chain is given an arbitrary displacement $y(x, 0) = y_0(x)$ and is released from rest. Establish the wave equation for this system and solve it to determine the displacement $y(x, t)$ at a later time t and describe the first two fundamental modes of vibration. Here x is the vertical distance measured from the free end of the chain and $y(x, t)$ is the displacement in the transverse direction. 15

7. Derive the heat conduction equation. Solve the equation to find the temperature $u(x, t)$ in a bar of length L, which is perfectly insulated, also at ends at $x = 0$ and $x = L$, i.e. : 15

$$\left.\frac{\partial u}{\partial x}\right|_{x=0} = \left.\frac{\partial u}{\partial x}\right|_{x=L} = 0$$

and

$$u(x, 0) = f(x).$$

8. Find solution to Laplace's equation in spherical coordinates which are independent of ϕ. Hence define surface harmonic and surface zonal harmonic. 15