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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **945**

Unique Paper Code : **222401** **E**

Name of the Paper : **Mathematical Physics—IV (PHHT-411)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **IV**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all taking at least *one* question from each Section.

All questions carry equal marks.

Section A

1. (*a*) Prove that the set I of all integers with the binary operation * defined by

$$a * b = a + b + 1$$

form a group. 7

Or

Determine whether or not W is a subspace of $\mathbf{R}^3$ where W consists of all vectors (a, b, c) in $\mathbf{R}^3$ such that :

$$a + b + c = 0.$$

(*b*) Find a basis and the dimension of solution space W of the following homogeneous system : 8

$$x + 2y - 2z + 2s - t = 0$$

$$x + 2y - z + 3s - 2t = 0$$

$$2x + 4y - 7z + s + t = 0.$$

2. (*a*) Linear Transformation T on $\mathbf{R}^3$ of all ordered triples of real numbers is defined by : 5

$$T\begin{bmatrix}1\\0\\0\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix},\quad T\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}3\\1\\2\end{bmatrix}\text{ and }T\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}2\\1\\3\end{bmatrix}$$

Compute :

$$T\begin{bmatrix}3\\-1\\4\end{bmatrix}.$$

(*b*) Linear transformation T on $\mathbf{R}^3$ is defined as :

$$T\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}2y+z\\3x-4y\\3x\end{bmatrix}.$$

Find the matrix representation of T relative to :

(*i*) the standard basis $\left\{e_1=\begin{bmatrix}1\\0\\0\end{bmatrix}, e_2=\begin{bmatrix}0\\1\\0\end{bmatrix}, e_3=\begin{bmatrix}0\\0\\1\end{bmatrix}\right\}$

(*ii*) the basis $\left\{\alpha_1=\begin{bmatrix}1\\1\\1\end{bmatrix}, \alpha_2=\begin{bmatrix}1\\1\\0\end{bmatrix}, \alpha_3=\begin{bmatrix}1\\0\\0\end{bmatrix}\right\}$. 10

Section B

3. (*a*) Verify Cayley-Hamilton theorem for the matrix :

$$A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}.$$

Hence find A^{-1}. 5

(*b*) If A is Hermitian matrix and B is skew-Hermitian matrix, show that :

$$A + iB \text{ and } A - iB$$

are Hermitian. 5

(*c*) If H is a Hermitian matrix and I is Identity matrix, show that :

$$(H - iI)\,(H + iI)^{-1}$$

is a unitary matrix :

$$\left(\text{here, } i = \sqrt{-1}\right).$$ 5

4. (*a*) Find the eigenvalues and eigenvectors of the matrix :

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix}.$$

Can A be diagonalized ? If yes, find a diagonalizing matrix P and verify that P diagonalizes A. 10

(*b*) Determine e^{A} if :

$$A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}.$$ 5

5. (*a*) Solve the coupled differential equations :

$$\ddot{y} = -4y + 2z$$

$$\ddot{z} = 3y - 3z$$

where, $y(0) = 1$, $z(0) = 4$, $\dot{y}(0) = \dot{z}(0) = 0$. 10

(*b*) Write the symmetric coefficient matrix of the following quadratic form :

$$x_1^2 + 2x_2^2 + 3x_3^2 + 4x_1x_2 + 5x_2x_3 + 8x_1x_3.$$ 5

Section C

6. Solve one-dimensional wave equation :

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad 0 \leq x \leq L; \ t > 0$$

under the boundary conditions $u(0, t) = 0$ and $u(L, t) = 0$ and initial conditions :

$$\left(\frac{\partial u}{\partial t}\right)_{t=0} = 0 \text{ and } u(x, 0) = \begin{cases} \frac{3h}{L} x, & 0 \leq x \leq \frac{L}{3} \\ \frac{3h}{2L}(L - x), & \frac{L}{3} \leq x \leq L \end{cases}$$

(here, c and h are constants). 15

7. Write down the two-dimensional heat equation in Cartesian coordinates and solve it for a uniform rectangular plate bounded by the lines $x = 0, y = 0; x = a, y = b$ when the initial temperature distribution is $f(x, y)$. The edges are kept at zero temperature and the faces of the plate are impervious to heat. 15

8. Write down three-dimensional Laplace's equation in cylindrical (ρ, ϕ, z) as well as in spherical (r, θ, ϕ) coordinates. Obtain the solution of Laplace's equation in spherical coordinates which are independent of ϕ. 15