[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 947 **E** **Your Roll No................**

Unique Paper Code : 235463

Name of the Course : **B.Sc. (Hons.) Physics**

Name of the Paper : Mathematics II (Analysis and Statistics) PHHT – 413

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt as per directions.

SECTION I

(Do any **two** *questions.)*

1. (a) If a series $\sum_{n=1}^{\infty} f_n$ converges uniformly to f in [a,b], where each f_n is continues in [a,b], then show that f is continues in [a,b].

   (b) Show that the sequence $\langle f_n \rangle$ of functions where $f_n(x) = x^n$ is uniformly convergent on [0, k], k < 1 and only pointwise convergent on [0, 1]. (6½,6)

2. (a) State Weierstrass M–Test for the uniform convergence of a series of functions defined on an interval I and test for uniform convergence the series

$$\sum_{n=1}^{\infty} \frac{\sin(x^2 + nx^2)}{n(n+1)} \text{ for all real x.}$$

   (b) Show that the sequence $\{f_n\}$, where

$$f_n(x) = \begin{cases} n^2 x & , 0 \le x \le \frac{1}{n} \\ -n^2 x + 2n & , \frac{1}{n} < x \le \frac{2}{n} \\ 0 & , \frac{2}{n} < x \le 1 \end{cases}$$

   is not uniformly convergent on [0,1]. (6½,6)

3. (a) Define sine function in terms of power series. Prove that

(i) $S(x + y) = S(x)C(y) + C(x)S(y)$

(ii) $C(x + y) = C(x)C(y) - S(x)S(y) \quad \forall\, x, y \in R$

Where C and S denote cosine and sine respectively.

(b) Determine the interval of convergence of the power series :

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-1)^n}{n}.$$

(6½,6)

SECTION II

4. Do any **three** parts :

(a) Prove the following result for Gamma function :

$$\sqrt{\pi}\,\Gamma(2m) = 2^{2m-1}\Gamma(m)\Gamma\left(m+\frac{1}{2}\right), \quad m > 0$$

(b) Show that : $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

(c) Prove that :

$$\int_0^{\infty} \frac{\tan^{-1}\alpha x}{x(1+x^2)}dx = \frac{\pi}{2}\log(1+\alpha), \quad \alpha \geq 0$$

assuming that differentiation under integral sign is valid.

(d) State Dirichlet's theorem for convergence of the integral of a product of two functions. Also, test the convergence of

$$\int_0^{\infty} \frac{\sin x}{x}\, dx$$

(e) Test the convergence of the improper integral :

$$\int_0^{\infty} e^{-x^2}\, dx$$

(5,5,5,5,5)

SECTION III

5. Do any **one** part : (5)

(a) The probability density function of a random variable X is

$$f(x) = \begin{cases} \frac{c}{\sqrt{x}} & 0 < x < 4 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Find value of c.

(ii) Find the distribution function.

(iii) Use the distribution function to determine $P\left(X < \frac{1}{4}\right)$ and $P(X > 1)$.

(b) If X takes on values 0, 1, 2 and 3 with probabilities $\frac{1}{125}, \frac{12}{125}, \frac{48}{125}, \frac{64}{125}$

(i) Find $E(X)$ and $E(X^2)$.

(ii) Use results of part (i) to determine value of $E((3X + 2)^2)$.

6. Do any **three** parts : (5,5,5)

(a) (i) Define a poisson random variable.

(ii) The average number of trucks arriving on any one day at a truck depot in a certain city is known to be 12. What is the probability that on a given day fewer than nine trucks will arrive at this depot ?

(b) (i) Determine the value of k for which the function given by

$$f(x, y) = kxy \begin{cases} x = 1, 2, 3 \\ y = 1, 2, 3 \end{cases}$$

can serve as a joint probability distribution.

(ii) Given $f(x, y) = \begin{cases} \frac{2}{3}(x + 2y) & 0 < x < 1; 0 < y < 1 \\ 0 & \text{elsewhere} \end{cases}$

find marginal densities of X and Y.

(c) Define :

(i) A normal random variable .

(ii) Moment Generating Function of a random variable.

Find the moment generating function of a normal random variable.

(d) Calculate the correlation coefficient for the following heights (in inches) of fathers (X) and their sons (Y)

X :	65	66	67	67	68	69	70	72
Y :	67	68	65	68	72	72	69	71

7. Do any **three** parts : (5,5,5)

(a) A simple sample of 1000 members is found to have a mean of 3.42 cm. Could it be reasonably regarded as a simple sample from a large population whose mean is 3.30 cm and standard deviation 2.6 cm ?

(b) A certain stimulus administered to each of the 12 patients resulted in the following increase in blood pressure 5, 2, 8, –1, 3, 0, –2, 1, 5, 0, 4, 6.

Can it be concluded that the stimulus will in general be accompanied by an increase in blood pressure ? ($t_{0.05}$ for 11 d.f. is 2.201)

(c) Two random samples drawn from two normal populations are :

Sample 1 :	20	16	26	27	23	22	18	24	25	19		
Sample 2 :	27	33	42	35	32	34	38	28	41	43	30	37

Obtain the estimates of the variances of the population and test whether the two populations have the same variance (F for 11 and 9 degrees of freedom at 5% level of significance = 3.11).

(d) Five coins are tossed 3200 times and the following results are obtained :

No. of heads :	0	1	2	3	4	5
Frequency :	80	570	1100	900	800	50

Test the hypothesis that the coins are biased.

($\chi^2_{0.05}$ f or 5 degree of freedom = 11.07)