[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 948 **E** **Your Roll No...............**

Unique Paper Code : 222403

Name of the Course : **B.Sc. (Hons.) Physics**

Name of the Paper : Numerical Analysis [PHHT–414]

Semester : IV

Duration : 3 Hours Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Question No. **1** is compulsory.
3. Attempt any **four** questions from the rest.
4. Scientific calculator (non-programmable) are allowed.

1. Attempt any **five** questions : (5×3=15)

(a) Explain why Newton-Raphson method is also called method of tangent ?

(b) Differentiate between round-off and truncation error. Give examples.

(c) Under what conditions do the Gauss Seidel method converge ?

(d) If $f(x) = ax$, show that $(E + E^{-1})f(x) = 2\,f(x)$, where E is shift operator.

(e) Given $\frac{dy}{dx} = -y$ with $y(0) = 1$. Using step size $h = 0.01$, find $y(0.01)$ by Euler Method.

(f) Give geometrical interpretation of Trapezoidal rule.

(g) What are forward and backward differences in a difference table ? How are they related ?

2. (a) Using bisection method determine the root of the equation $y = x^3 - x^2 - 2x + 1$. Perform four iterations and tabulate for $x \in [-2,0]$.

(b) Solve following equations using Gauss-elimination method :

$$x_1 + 2x_2 + x_3 = 1$$
$$2x_1 - x_2 + 2x_3 = -3$$
$$x_2 + 3x_3 = 1$$

(c) Given actual value of $x = 5/3$ and $y = 1/3$. Use five digit floating point representation to write floating form of x & y. Calculate relative and absolute error for

(i) $x + y$ and (ii) $x \times y$. (3×5=15)

3. (a) Solve the following by Gauss Seidel method (Perform only three iterations with initial guess as $x_0 = 0$, $y_0 = 0$, $z_0 = 0$)

$$27x + 6y - z = 54$$
$$6x + 15y + 2z = 72$$
$$x + y + 54z = 110$$

(b) A particle is moving along straight line. The displacement (x) at some time (t) given below

Time (t) sec	**Displacement (x) km**
0	5
1	8
2	12
3	17
4	26

Find the velocity and acceleration at t = 4 sec.

(c) Using Iterative method, find the dominant eigenvalue and the corresponding eigenvector of the following matrix

$$\begin{bmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 3 \end{bmatrix} \quad \textit{with initial eigenvector} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$ (3×5=15)

4. (a) If $[x_1, x_2, x_3]$ represents divided difference of second order then show that

$$[x_1, x_2, x_3] = \frac{y_1}{(x_1 - x_2)(x_1 - x_3)} + \frac{y_2}{(x_2 - x_1)(x_2 - x_3)} + \frac{y_3}{(x_3 - x_1)(x_3 - x_2)}$$

(b) Using interpolation find the points of maxima and minima for the following data

x	y
−1	6
1	−4
3	10

(c) A curve is drawn to pass through the points given by the following table

x	y
1	2
1.5	2.4
2	2.7
2.5	2.8
3	3
3.4	3.6
4	2.4

Find $\int_2^4 y\,dx$ using (i) Trapezoidal rule (ii) Simpson's 1/3 rule. (3×5=15)

5. (a) Deduce Newton's backward difference interpolation formula.

(b) Find the solution of following differential equation using second order Runge-Kutta method

$$\frac{dy}{dx} + 2xy^2 = 0$$

$y(0) = 1$ with $h = 0.5$ on interval $[0,1]$.

(c) Fit the following data to a quadratic function :

x	y
0	10
2	15
4	18
6	25

(3×5=15)

6. (a) Using Gram-Schmidt Orthogonalization process obtain first four orthogonal polynomial fi(x) on $[-1,1]$ with respect to weight $w(x) = 1$.

(b) Explain the concept and criteria of least square fit. Derive the normal equations for general polynomial for degree 'm' to a given set of 'n' data points.

(c) Evaluate the integral

$$\int_1^2 \frac{1}{(1+x)} dx$$

using Gauss Legendre's three-point formula. Given the nodes and corresponding weights as

n	u_i	w_i
3	0	0.88889
	∓ 0.7746	0.55556

(3×5=15)