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Your Roll No...............

1227

B.Sc. (Hons.)/III **A**

PHYSICS—Paper XVII

(Mathematical Physics—III)

Time : 3 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all.

Question No. 1 is compulsory.

Attempt *one* question from each Section.

1. Attempt any *five* of the following : 5×2

(*a*) Let $V = R^3$. Determine if W is a subspace of V where

$W = \{(a, b, c) : a + b + c = 0\}$.

(*b*) Let V be the vector space of functions from R into R. Show that $f, g, h \in V$ are independent where

$f(t) = \sin t$, $g(t) = \cos t$, $h(t) = t$.

(*c*) If H is a Hermitian matrix, prove that e^{iH} is unitary.

(*d*) If $F(\alpha)$ is the Fourier transform of $f(t)$, find the Fourier transform of $\{f(t) \cos at\}$.

(*e*) Prove that

$$x\delta'(x) = -\delta(x)$$

where $\delta(x)$ is the Dirac Delta function.

(*f*) Show that there is no isotropic tensor of order one except the null vector.

(*g*) Prove that

$$\left\{\int_0^t F(u)\, du\right\} = \frac{f(s)}{s}$$

where $f(s) = L\{F(t)\}$.

Section A

2. (*a*) Find the dimension and a basis of the solution space W of the homogenous system :

$$x + 2y + 2z - s + 3t = 0$$

$$x + 2y + 3z + s + t = 0$$

$$3x + 6y + 8z + s + 5t = 0$$

(*b*) Let $T : R^2 \rightarrow R^2$ be defined by

$$T(x, y) = (2x - 3y, x + 4y).$$

Find the matrix of T w.r.t. basis $\{f_1 = (1, 3), f_2 = (2, 5)\}$ of R^2. 4,3

3. (*a*) Let W be the subspace of R^4 generated by the vectors $(1, -2, 5, -3)$, $(2, 3, 1, -4)$ and $(3, 8, -3, -5)$.

(*i*) Find a basis and dimension of W.

(*ii*) Extend the basis of W to a basis of the whole space R^4.

(*b*) Let T be the operator on R^3 defined by

$$T(x, y, z) = (2x, 4x - y, 2x + 3y - z).$$

(*i*) Show that T is invertible;

(*ii*) Find a formula for T^{-1}. 4,3

Section B

4. (*a*) Using the matrix method, solve the following system of equations :

$$\dot{x}_1 = 2x_1 + 2x_2 + x_3$$

$$\dot{x}_2 = x_1 + 3x_2 + x_3$$

$$\dot{x}_3 = x_1 + 2x_2 + 2x_3$$

with the initial conditions :

$$x_1(0) = x_2(0) = 2, \text{ and } x_3(0) = 1.$$

(*b*) Show that eigenvalues of a unitary matrix are unimodular. 5,2

5. (*a*) Find A^{-1} using Caley-Hamilton theorem for the matrix A where

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$

(*b*) Find the eigenvalues and eigenvectors of the matrix :

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}.$$ 4,3

Section C

6. (*a*) Given

$$T_{mn} = \begin{pmatrix} -xy & -y^2 \\ x^2 & xy \end{pmatrix}$$

Show whether T_{mn} is a tensor or not. Verification may be done for only one of the components.

(*b*) If T_{ij} is a skew-symmetric tensor of order two, prove that

$$(\delta_{ij}\delta_{lk} + \delta_{il}\delta_{jk})T_{ik} = 0.$$

(*c*) Show that

$$\epsilon_{ijk} A_j A_k = 0 .$$ 4,2,1

7. (*a*) Using tensor methods, verify the identity

$$\vec{\nabla}(\vec{A} \cdot \vec{B}) = \vec{A} \times (\vec{\nabla} \times \vec{B}) + \vec{B} \times (\vec{\nabla} \times \vec{A}) + (\vec{A} \cdot \vec{\nabla})\vec{B} + (\vec{B} \cdot \vec{\nabla})\vec{A}.$$

(*b*) Show that we can associate a vector with any antisymmetrical tensor of order two.

(*c*) Show that gradient of a scalar function is a tensor of order one. 3,2,2

Section D

8. (*a*) Using convolution theorem for Fourier transform, solve :

$$\int_{-\infty}^{\infty} \frac{y(u)\,du}{(x-u)^2 + a^2} = \frac{1}{x^2 + b^2}, \; 0 < a < b.$$

(*b*) Solve :

$$y'' + y = t, \; y(0) = 1, \; y'(0) = -2.$$

by method of Laplace transforms. 4,3

9. (*a*) An infinitely long string having one end at $x = 0$ is initially at rest on x-axis. The end $x = 0$ undergoes a periodic transverse displacement given by $A_0 \sin \omega t$, $t > 0$. Find the displacement of any point on the string at time t.

(*b*) Find the Fourier transform of slit function $f(x)$ defined as :

$$f(x) = \begin{cases} 1/\in & |x| \leq \in \\ 0 & |x| > \in \end{cases}$$ 5,2