This question paper contains 2 printed pages. *Your Roll No. ....*

Sl. No. Of Ques. Paper	:	**8420C**
Unique Paper Code	:	**222501**
Name of the Paper	:	**PHHT- 515 : Mathematical Physics - V**
Name of the Course	:	**B.Sc. (Hons) Physics Part III**
Semester	:	**V**
Duration	:	**3 hours**
Maximum Marks	:	**75**

Do five questions in all. All questions carry equal marks. Question No. 1 is compulsory. Do two questions from each Section.

Q1. Do any *five* : (15)

a) Fourier Transform of a Gaussian function is a Gaussian function.

b) Find $L(F(t))$ where

$F(t) = \cos\left(t - \frac{2\pi}{3}\right) u\left(t - \frac{2\pi}{3}\right)$ and

$u\left(t - \frac{2\pi}{3}\right)$ is the unit step function.

c) Find $L(t^n)$ for n as a positive integer.

d) Show that $L(\delta(t)) = 1$ where $\delta(t)$ is the Dirac Delta function.

e) Show that div (curl F) = 0 using tensors.

f) Show that velocity and acceleration are contravariant vectors.

g) Show that $\vec{A} \times \vec{B}$ transforms like tensor of rank one.

Section A

Q2. (10+5)

a) Find the Fourier sine transform of

$f(t) = e^{-pt}$ $p > 0$ and

Evaluate the integral

$$\int_0^\infty \frac{\omega \sin \omega t}{\omega^2 + p^2} d\omega$$

b) Show that $\int_{-\infty}^{\infty} f(x)\delta'(x)dx = -f'(0)$.

Q3. (5+5+5)

a) Prove that

$$L\left(\frac{1}{t} f(t)\right) = \int_s^\infty f(s)ds$$

b) Using convolution theorem for Laplace transforms, find

$$L^{-1}\left(\frac{1}{s(s^2 + a^2)}\right)$$

c) Show that the derivative of unit step function is Dirac Delta Function.

Q4. (12+3)

a) Solve the given coupled differential equations using Laplace Transforms

$$\frac{dx}{dt} = 2x - 3y$$
$$\frac{dy}{dt} = y - 2x$$

subject to initial conditions $x(0) = 8,\ y(0) = 3.$

b) Find out

$$L\left(\int_0^t \frac{\sin u}{u}\, du\right)$$

Section B

Q5. (5+10)

a) Given vector

$$\vec{U} = (x, \quad x + y, \quad x + y + z)$$

Find the second order anti-symmetric tensor associated with it.

b) Show that

$$\epsilon_{ijk}\, \epsilon_{lmn} = \begin{vmatrix} \delta_{il} & \delta_{im} & \delta_{in} \\ \delta_{jl} & \delta_{jm} & \delta_{jn} \\ \delta_{kl} & \delta_{km} & \delta_{kn} \end{vmatrix}$$

Hence prove

$$\epsilon_{ijk}\, \epsilon_{ijk} = 6$$

Q6. (10+5)

a) Define the Pure Strain Tensor e_{ij}. Establish that it is a symmetric tensor of order 2. Also give the physical significance of its components e_{11} and e_{12}.

b) Define Quotient Law.
Let $A(i, j, k)$ be a set of N^3 functions whose inner product with an arbitrary tensor B^{jk} yields a tensor C^i. What can you conclude about $A(i, j, k)$?

Q7. (12+3)

a) The length ds of a line element in a 2-dimensional suface $\theta, \emptyset$ is given by
$$ds^2 = R^2\, d\theta^2 + R^2 sin^2\theta d\emptyset^2 \qquad \text{with R = constant.}$$
Find all the components of the metric tensor $g_{\mu\nu}$ and the Christoffel symbols of first kind for this surface.

b) Show that $A^{\mu}\, B_{\mu}$ is invariant.