This question paper contains 3 printed pages. *Your Roll No. ....*

Sl. No. Of Ques. Paper	:	**8421C**
Unique Paper Code	:	**222502**
Name of the Paper	:	**PHHT-516 : Quantum Mechanics**
Name of the Course	:	**B.Sc. (Hons.) Physics Part III**
Semester	:	**V**
Duration	:	**3 hours**
Maximum Marks	:	**75**

Attempt five questions in all. Question No. 1 is compulsory.

Use of non-programmable scientific calculators is allowed.

Symbols have their usual meaning.

Q. 1 *Attempt any five of the following.*

a) Starting from Heisenberg's Uncertainty Principle $\Delta x \Delta p \geq \hbar/2$, obtain a similar inequality for the variables x and λ, where λ is the de Broglie wavelength.

b) The wave function for a particle confined in a one-dimensional box of length L is given by

$$\psi(x) = A\sin(n\pi x / L).$$

Normalize the wave function and evaluate the expectation values of its momentum.

c) Determine which of the following wave functions are physically acceptable solutions of the Schrodinger wave equation:

$$(1)\ \tan x \quad (2)\ \sin x \quad (3)\ 1/x \quad (4)\ \exp(-x^2/2) \quad (5)\ \sec x \quad (6)\ \exp(ikx)$$

d) An X-ray photon undergoes Compton scattering by 90^0. If the frequencies of the incident and the scattered photons are υ and υ' respectively, calculate the de Broglie wavelength associated with the recoil electron.

e) The uncertainty in the velocity of a particle is equal to its velocity. Calculate the minimum uncertainty in the position of the particle in terms of its de Broglie wavelength.

f) Calculate the group velocity of ocean waves whose phase velocity is given by

$v_p = \sqrt{\dfrac{g\lambda}{2\pi}}$, where λ is wavelength of ocean waves and g the acceleration due to gravity.

g) Explain what is meant by space-quantization of angular momentum L. What role does the magnetic quantum number m_l play in this quantization?

h) Let an electron be regarded as a linear harmonic oscillator with angular frequency 5×10^{14}/sec . Calculate: (i) its zero-point energy and (ii) the classical limits of its motion in the n = 1 state.

(3 x 5)

Q. 2 a) Why is the classical wave theory unable to explain the observations of the photoelectric effect? How does Einstein's photoelectric equation resolve these difficulties?

b) Show that a free electron cannot completely absorb a photon and conserve both energy and momentum.

c) The stopping potential for photoelectrons emitted from a surface illuminated by light of wave length $\lambda = 4900\ A^0$ is 0.7 V. When the incident light is changed, the stopping potential changes to 1.41 V. What is the new wavelength?

(8, 3, 4)

Q. 3 a) Explain why it is necessary to create a wave packet to describe a particle in quantum mechanics. Construct a wave packet using two simple harmonic waves and obtain Heisenberg's uncertainty principle connecting position and momentum of a particle.

b) Show that the Uncertainty principle can be used to derive an expression for the radius of the first Bohr orbit of the Hydrogen atom.

(2, 7, 6)

Q. 4 a) Explain the de Broglie hypothesis for matter waves. Describe the Davisson Germer experiment in detail and point out how it established the wave nature of electrons.

b) Calculate the de Broglie wave length for an electron with kinetic energy of

i) 1 eV and ii) 1 MeV.

(Rest energy of an electron = 0.511 Mev)

(10, 5)

Q. 5 a) Write down the time dependent one-dimensional Schrödinger equation for a free particle of mass m. Show that the states with definite energy E can be represented by the wave function $\Psi(x,t) = \Phi(x) \exp(-iEt/\hbar)$, where $\Phi(x)$ is the solution of the time independent Schrödinger equation.

b) Obtain the energy eigenvalues and the normalized wave functions for a free particle of mass m trapped in a one dimensional box of length L

(8, 7)

Q. 6 a) A beam of particles, each of mass m and energy E, moving along the x-axis is incident on a regular potential barrier of width a and height V_0 given by

$$
\begin{aligned}
V(x) &= 0 \qquad x < 0 \\
&= V_0 \qquad 0 < x < a \\
&= 0 \qquad x > a
\end{aligned}
$$

Obtain an expression for the transmission coefficient (T) for the case $E < V_0$. Hence derive the limiting expression for T for a very broad, high barrier.

b) What do you understand by tunnel effect? Explain with reference to the Tunnel diode.

(12, 3)

Q. 7 a) Set up the time-dependent Schrodinger equation for a linear harmonic oscillator and obtain an expression for the energy eigenvalues of the oscillator. Draw the energy level diagram.

b) Comment on the statement that: "Zero point energy of a harmonic oscillator is in agreement with the Uncertainty Principle."

(12, 3)

Q. 8 a) Using the radial equation for hydrogen atom

$$\frac{d^2R}{dr^2} + \frac{2}{r}\frac{dR}{dr} + \frac{2m}{\hbar^2}\left\{E + \frac{e^2}{4\pi\varepsilon_0 r} - \frac{l(l+1)\hbar^2}{2mr^2}\right\}R = 0,$$

obtain an expression for the energy eigenvalues.

b) The ground state wave function for hydrogen is

$$\psi(r) = \frac{1}{\sqrt{\pi a_0^3}} e^{-r/a_0}.$$

Calculate the probability of finding the electron at a distance less than a_0

(10, 5)

Physical Constants:

$h = 6.63 \times 10^{-34}$ Js

$e = 1.6 \times 10^{-19}$ C

$m_e = 9.1 \times 10^{-31}$ kg