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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6215**

Unique Paper Code : **222501** **D**

Name of the Paper : **Mathematical Physics—V(PHHT-515)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do *five* questions in all. Question No. **1** is compulsory.

Do *two* questions from each Section.

1. Answer any *five* questions :

(*a*) If $F(k)$ is the Fourier transform of $f(t)$, find the Fourier transform of $f(t)\cos at$.

(*b*) Find the Fourier transform of $f(t) = e^{-|t|}$.

(*c*) Show that the Laplace transform of a periodic function $f(t)$ is :

$$L\{f(t)\} = \frac{\int_0^T f(t)e^{-st}dt}{(1 - e^{-sT})} \quad \text{where } f(t + T) = f(t),\ s > 0.$$

P.T.O.

(*d*) Show that :

$$L^{-1}\left\{\frac{F(s)}{s}\right\} = \int_0^t f(u)du$$

where

$$F(s) = L\{f(u)\}.$$

(*e*) Prove that :

$$\int_{-\infty}^{\infty} \delta(x-a)\delta(x-b)dx = \delta(a-b).$$

(*f*) Show that we can associate an anti-symmetric tensor of order two (w_{lm}) with a given vector u_k and represent w_{lm} in the form of a matrix.

(*g*) Prove that g_{ij} is a covariant tensor of rank two. 3×5=15

Section A

2. (*a*) State the Convolution theorem for Laplace transforms. Using this theorem, evaluate

$$L^{-1}\left\{\frac{1}{s^2(s+1)^2}\right\}.$$

(*b*) Find the Fourier cosine and sine transforms of $f(x)$ where $f(x) = e^{-x}$, $x > 0$. 7,8

3. (*a*) Solve the following simultaneous differential equations using Laplace transforms :

$$\frac{dx}{dt} + x + y = 0$$

$$\frac{dy}{dt} + 4x + y = 0$$

Given :

$$x(0) = y(0) = 1.$$

(b) Determine :

$$L^{-1}\left\{\frac{3s+1}{(s-1)(s^2+1)}\right\}.$$ 8,7

4. (a) Find the Fourier sine and cosine transforms of $f(t) = e^{-pt}$, $p \geq 0$. Using these results, evaluate the following integrals :

$$\int_0^\infty \frac{\cos kt}{p^2+k^2}\,dk \text{ and } \int_0^\infty \frac{k \sin kt}{p^2+k^2}\,dk.$$

(b) Evaluate using Laplace transforms $\int_0^\infty te^{-3t} \sin t \, dt.$ 10,5

Section B

5. (a) Prove that the moment of inertia is a symmetric tensor of order two. Represent it in the form of a matrix.

(b) The length ds of a line element in cylindrical coordinates is expressed as :

$$ds^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2$$

Determine the metric tensor g_{pq} and express it as a matrix. 10,5

6. (a) Using tensors prove the following identity :

$$(A \times B) \times (C \times D) = B[A.(C \times D)] - A[B.(C \times D)].$$

(b) Show that the only second order isotropic tensor is Kronecker delta δ_{lm}. 10,5

7. (*a*) Given $ds^2 = (dr)^2 + r^2(d\theta)^2 + r^2 \sin^2\theta(d\phi)^2$, calculate the values of the following Christoffel symbols :

(*i*) $[2\ 2,\ 1]$ and $[1\ 3,\ 3]$

(*ii*) $\begin{Bmatrix} 1 \\ 2\ 2 \end{Bmatrix}$ and $\begin{Bmatrix} 3 \\ 1\ 3 \end{Bmatrix}$.

(*b*) What are pseudo tensors ? Using examples compare the behaviour of a tensor and a pseudo tensor. Give an example of a third order pseudo tensor. 10,5