This question paper contains **4+2** printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **6216**

Unique Paper Code : **222502** **D**

Name of the Paper : **Quantum Mechanics (PHHT-516)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all. Question No. **1** is compulsory.

Use of non-programmable scientific calculator is allowed.

Symbols have their usual meaning.

1. Attempt any *five* of the following : 3×5=15

(*a*) A metal surface when irradiated with light of wavelength 5896 Å emits electrons for which the stopping potential is 0.12 V. When the same surface is irradiated with 2830 Å, it emits electrons for which the stopping potential is 2.20 V. Calculate the value of Planck's constant.

P.T.O.

(*b*) Compare the de Broglie wavelengths for an electron with a kinetic energy of 1 eV and a ball of mass 300 gm travelling at 100 km/hr.

(*c*) Determine the smallest possible uncertainty in the position of an electron moving with velocity 3×10^7 m/s.

(*d*) Establish time independent form of Schrödinger equation for stationary states.

(*e*) Determine the probability of finding a particle of mass m between $x = 0$ and $x = L/10$, if the particle is described by the normalized wave function :

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

for $0 \le x \le L$ and is in the $n = 3$ state.

(*f*) A radial function in spherical polar coordinates is :

$$R_n(r) = Ce^{-r/2}\, U_n(r),$$

where C is a normalization constant. Discuss the physical acceptability of $R_n(r)$ if $U_n(r)$ behaves as :

(*i*) $1/r^2$ for small values of r and as a polynomial in r otherwise; and

(*ii*) a polynomial in r of degree more than 3 for all values of r.

(*g*) A one-dimensional harmonic oscillator is in a state described by the wave function :

$$\psi(x, 0) = \frac{1}{2}\psi_0(x) + \frac{i}{\sqrt{2}}\psi_1(x) + \frac{1}{\sqrt{2}}e^{i\pi/3}\psi_2(x)$$

where $\psi_n(x)$ are the usual normalized orthogonal wave functions. Normalize the wave function $\psi(x, 0)$.

(*h*) Find the classical amplitude of a one-dimensional harmonic oscillator in its ground state with an energy $\frac{1}{2}\hbar\omega$.

2. (*a*) In the Compton scattering of a photon of frequency ν by a free electron through an angle □, using the expressions for momentum conservation : 10

$$pc\cos\theta = h\nu - h\nu'\cos\phi \text{ and } pc\sin\theta = h\nu'\sin\phi$$

and the expression for change in wavelength of scattered photon :

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - \cos\phi)$$

prove that :

$$\tan\theta = \frac{1}{[1+\beta]\tan\frac{\phi}{2}}; \text{ where } \beta = \frac{h\nu}{m_o c^2}$$

(*b*) A metal surface emits electrons with maximum kinetic energies E_1 and E_2 when illuminated with light of wavelengths λ_1 and λ_2 respectively, where $\lambda_1 > \lambda_2$. Prove that the Planck's constant h and work function ϕ of the metal are given by : 5

$$h = \frac{(E_2 - E_1)\lambda_1 \lambda_2}{c(\lambda_1 - \lambda_2)} \text{ and } \phi = \frac{E_2 \lambda_2 - E_1 \lambda_1}{(\lambda_1 - \lambda_2)}.$$

3. (*a*) Explain de Broglie hypothesis for matter waves. Show that for the de Broglie wave associated with a moving particle the group velocity is equal to the particle velocity. 5

(*b*) Assume that at time $t = 0$, a single non-interacting electron is located near $x = x_0$ with the probability P dx of finding it between x and $x + dx$ being given by :

$$\psi(x, 0) = Ae^{-(x-x_0)^2/2a^2} e^{ip_0 x/\hbar}$$

Obtain the expectation values of x and p. Also show that : 10

$$\Delta x \,.\, \Delta p = \hbar/2.$$

4. (*a*) Describe an experiment to locate the position of a free electron by a microscope using γ ray and hence, obtain an expression for uncertainty principle. 10

(*b*) Determine the minimum uncertainty in the position of a particle in terms of de Broglie wavelength when the uncertainty in the velocity of a particle in one-tenth of its velocity. 5

5. (*a*) Obtain the energy eigen values and the normalized wave functions for a free particle of mass m trapped in a one-dimensional box of length L along x-axis in positive direction of x from the origin. 10

(*b*) The wave function of a particle confined in a box of linear dimension L along x-axis is :

$$\psi(x) = Ae^{i\alpha x};\ 0 \leq x \leq L.$$

Find the probability of finding the particle in the distance $0 \leq x \leq \frac{L}{4}$. 5

6. Solve the time independent Schrödinger equation for the energy levels of a one-dimensional harmonic oscillator. Draw the energy level diagram. Explain the physical significance of zero-point energy. 15

7. A particle of mass m and energy E moves along x-axis from a region of zero potential towards a one-dimensional step potential barrier of height V_0 of infinite extent. Assuming $E > V_0$, derive expressions for the reflection and transmission coefficients. Comment on the wavelengths associated with the incident, reflected and transmitted waves. Also, obtain expressions for probability current densities associated with the incident, reflected and transmitted waves. 15

8. (*a*) Solve the angular equation of hydrogen atom in spherical polar coordinates, given as :

$$\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial y}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2 y}{\partial\phi^2}+\lambda y=0$$

to obtain relation $\lambda = l(l+1)$. 11

(*b*) Given the wave function of ground state of hydrogen atom is :

$$\psi_{100}(r)=\frac{e^{-r/a_0}}{(\pi a_0^3)^{1/2}},$$

where the symbols have usual meaning and :

$$a_0=\frac{4\pi\epsilon_0\hbar^2}{me^2}.$$

Calculate the most probable distance of electron from nucleus in ground state. 4

$\hbar = 1.054 \times 10^{-34}$ Js

$h = 6.63 \times 10^{34}$ Js

Rest mass of electron $= 9.1 \times 10^{-31}$ kg

Charge of electron $= 1.6 \times 10^{-19}$ C

Rest mass energy of electron = 512 KeV

Velocity of light in free space $= 3 \times 10^8$ m/s.