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Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **852**

Unique Paper Code : **222501** **G**

Name of the Paper : **Mathematical Physics—V (PHHT-515)**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Do *five* questions in all.

Question No. **1** is compulsory.

Do **2** questions from each Section.

1. Do any *five* questions :

(*a*) If F(*k*) is the Fourier transform of $f(x)$, find the Fourier transform of $f(ax)$.

(*b*) Find the Fourier transform of $e^{-|t|}$.

(*c*) If

$$L\{F(t)\} = f(s),$$

find $L\left\{F\left(\frac{t}{a}\right)\right\}$.

(*d*) Find the Laplace transform of the Dirac delta function $\delta(t-a)$, where a is positive real constant.

P.T.O.

(*e*) Show that :

$$\delta'(x) = -\frac{\delta(x)}{x}.$$

(*f*) Show that gradient of a scalar function is a tensor of order one.

(*g*) Find the second order anti-symmetric tensor associated with the vector :

$$2\hat{i} - 3\hat{j} + \hat{k}\cdot$$

(*h*) Prove that product of tensors of rank one is a tensor of rank two. 5×3=15

Section A

2. (*a*) Find Fourier sine transform of e^{-x} and hence prove that : 4,4

$$\int_0^\infty \frac{t\sin tx}{1+x^2}\,dt = \frac{\pi}{2}e^{-x}.$$

(*b*) State and prove Convolution theorem for Fourier transforms. 2,5

3. (*a*) Find : 5

$$L^{-1}\left\{\frac{1}{s^2(s^2+a^2)}\right\}.$$

(*b*) Using Laplace transforms, solve the following coupled differential equations :

$$\frac{dX}{dt} + Y = 0, \qquad \frac{dY}{dt} - X = 0$$

under the condition X(0) = 1, Y(0) = 0. 10

4. (*a*) For the function :

$$G(t) = \begin{cases} e^{-xt}\phi(t) & ; t < 0 \\ 0 & ; t > 0 \end{cases}$$

Find the relation between Fourier transform of G(*t*) and Laplace transform of $\phi(t)$. 6

(*b*) If F(*t*) is a periodic function of period T, find its Laplace transform. 5

(*c*) Prove that :

$$\delta(ax) = \frac{\delta(x)}{|a|},$$

where $a > 0$. 4

Section B

5. (*a*) Derive an expression for the Moment of Inertia tensor. Prove that it is a symmetric tensor and it transforms like a second order tensor. 5,2,3

(*b*) Show that : 5

$$\epsilon_{iks}\,\epsilon_{mps} = \delta_{im}\,\delta_{kp} - \delta_{ip}\,\delta_{km}.$$

6. (*a*) Define Kronecker-Delta function. Show that it is : 1,2,2

(*i*) an isotropic tensor

(*ii*) a symmetric tensor of order 2.

(*b*) Using tensors, prove the following identities : 5,5

(*i*) $\vec{\nabla} \times \left(\phi \vec{A}\right) = \phi\left(\vec{\nabla} \times \vec{A}\right) + \left(\vec{\nabla}\phi\right) \times \vec{A}$

(*ii*) $\vec{\nabla} \times \left(\vec{\nabla} \times \vec{A}\right) = \vec{\nabla}\left(\vec{\nabla} . \vec{A}\right) - \nabla^2 \vec{A}.$

7. (*a*) If

$$ds^2 = 3(dx^1)^2 + 5(dx^2)^2 - 4dx^1dx^2,$$

find the matrices :

(*i*) (g_{ij}),

(*ii*) (g^{ij}), and

(*iii*) the product of (g_{ij}) and (g^{ij}). 2,4,2

(*b*) Prove that : 7

$$\left\{ \begin{matrix} p \\ p\ q \end{matrix} \right\} = \frac{\partial}{\partial x^q} \ln \sqrt{g}.$$