This question paper contains 4+1 printed pages]

Roll No. | | | | | | | | | | | |

S. No. of Question Paper : **853**

Unique Paper Code : **222502** **G**

Name of the Paper : **PHHT-516 : Quantum Mechanics**

Name of the Course : **B.Sc. (Hons.) Physics**

Semester : **V**

Duration : **3 Hours** Maximum Marks : **75**

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *five* questions in all.

Question No. **1** is compulsory.

Use of non-programmable scientific calculator is allowed.

Symbols have their usual meaning.

1. Attempt any *five* questions : 5×3=15

(*i*) A particle of mass m is confined to a one-dimensional line of lenght L. From arguments based on the wave interpretation of matter, show that the energy of the particle is quantized.

(*ii*) What is stationary in stationary states of time-independent Schrödinger equation ?

(*iii*) Determine the speed of an electron whose de-Broglie wavelength is equal to its Compton wavelength.

(*iv*) Which is more effective in preventing tunnelling, the barrier potential height (U) or the barrier width (L) ? Why ?

(*v*) A particle of rest mass m_0 is moving uniformly in a straight line with relativistic velocity, βc, where c is the velocity of light in vacuum and $0 < \beta < 1$. What is the phase velocity of the de-Broglie wave associated with the particle ?

(*vi*) What are the possible values of n, l and m_l for $3p$ electron in a hydrogen atom ?

(*vii*) A free particle of mass m is described by the wave function

$$\psi(x) = Ae^{i\mu x}, \text{ A and } \mu \text{ are constants.}$$

Determine the momentum of the particle.

(*viii*) Differentiate between Photoelectric effect and Compton effect.

2. (*a*) Write the conclusion of each of the following experiments : 3

(*i*) Photoelectric experiment

(*ii*) Davisson-Germer experiment

(*iii*) Franck-Hertz experiment.

(*b*) The work function of a metal is 4.14 eV. What is the maximum wavelength of a photon that can eject an electron from the metal ? 3

(*c*) A photon of frequency v is scattered (at an angle ϕ) by an electron initially at rest. Show that the maximum kinetic energy of the recoil electron is given by

$$K_{max} = \frac{2hv\xi}{1+2\xi}$$

where $\xi = \frac{hv}{m_0c^2}$, m_0 is the rest mass of electron. 9

3. (*a*) Show that the phase velocity of the de-Broglie waves of a particle of rest mass m_0 and de-Broglie wavelength λ is given by 7

$$v_p = c\sqrt{1+\left(\frac{m_0c\lambda}{h}\right)^2}$$

(*b*) If the above particle has kinetic energy K, then prove that the expressions for the de-Broglie wavelength of this particle are given by :

(*i*) $\lambda = \frac{h}{\sqrt{2m_0 K}}$ (non-relativistic case) 3

(*ii*) $\lambda = \frac{ch}{\sqrt{K(K + 2m_0 c^2)}}$ (relativistic case) 5

4. (*a*) A free particle of mass m moving in one-dimension (say along positive x-axis) with momentum p and energy E can be described in quantum mechanics by the monochromatic plane wave $\psi(x, t) = Ae^{i(px - Et)/\hbar}$, where A is some constant :

(*i*) Obtain the time-dependent Schrödinger equation satisfied by this free particle. 7

(*ii*) How can we normalize the above wave function $\psi(x, t)$? 2

(*b*) Suppose $\psi_1(x)$ and $\psi_2(x)$ are the solutions of one-dimensional time-dependent Schrödinger equation then prove that

$$\psi(x) = a_1\psi_1(x) + a_2\,\psi_2(x)$$

is also a solution. Here a_1 and a_2 are complex constants. 6

5. (*a*) A particle of mass m and energy E moves inside an infinite potential well :

$$V = \begin{cases} 0, 0 < x < L \\ \infty, x < 0 \text{ and } x > L \end{cases}$$

The normalized wave functions of this particle are given by

$$\psi_n(x) = \sqrt{\frac{2}{L}} \sin\frac{n\pi x}{L}, 0 \le x \le L \text{ and } n = 1, 2, 3, \ldots$$

Prove that the expectation values $\langle xp_x \rangle$ and $\langle p_x x \rangle$ in the nth state are related by

$\langle xp_x \rangle_n - \langle p_x x \rangle_n = i\hbar$. 6

(*b*) Using $\psi_n(x)$, obtain momentum eigenfunctions and momentum eigenvalues of this particle. 5

(*c*) Find the probability of finding this particle in the range $0 \le x \le L/n$ when it is in the n^{th} state.

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6. One-dimensional harmonic oscillator (mass = m and angular frequency = ω) is in the ground state given by

$$\psi_0(x) = \left(\frac{\beta^2}{\pi}\right)^{1/4} \exp\left(\frac{-\beta^2 x^2}{2}\right), -\infty \le x \le \infty \text{ and } \beta = \sqrt{\frac{m\omega}{\hbar}}$$

(*a*) Prove that in this state

$$\Delta x \, \Delta p_x = \frac{\hbar}{2}$$

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$$\left[\text{Note} : \Delta y = \sqrt{\langle y^2 \rangle_0 - \langle y \rangle_0^2}, \langle f \rangle_0 = \int_{-\infty}^{\infty} \psi_0^*(x) f \psi_0(x) dx, \; p_x = -i\hbar \frac{\partial}{\partial x}\right.$$

$$\left. \text{and } \int_0^{\infty} u^2 e^{-u^2} du = \frac{\sqrt{\pi}}{4} \right]$$

(*b*) Determine the probability of finding this harmonic oscillator in the classically forbidden region.

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7. (*a*) The radial Schrödinger equation for hydrogen atom is given as 12

$$\frac{d^2u}{dr^2} + \frac{2m}{\hbar^2}\left[E - V - \frac{\hbar^2 l(l+1)}{2mr^2}\right] u = 0$$

where $u(r) = r\,R(r)$ and $V(r) = \dfrac{-e^2}{4\pi\varepsilon_0 r}$

Prove that the radial wave function, $R_{nl}(r)$ is given by

$$R_{nl}(r) = A e^{-\gamma r/2} (\gamma r)^l L_{n+l}^{2l+1}(\gamma r), \; n = 1, 2, 3, \ldots$$

where, $\gamma = \frac{2}{na_0}$, $a_0 = \frac{4\pi\varepsilon_0\hbar^2}{me^2}$ is Bohr radius and A is some constant.

[Note : Solution of $y\frac{d^2h}{dy^2} + (2l + 2 - y)\frac{dh}{dy} + (n - l - 1)h = 0$, is given by

$h(y) = CL_{n+l}^{2l+1}(y)$, C is some constant]

(*b*) Verify that the most probable value of r for $1s$ electron in a hydrogen atom is equal to a_0. 3

$$\left[\text{Given} : R_{10}(r) = \frac{2}{a_0\sqrt{a_0}}e^{-r/a_0}\right]$$

8. (*a*) A particle of mass m and energy E moves in a finite potential well :

$$V = \begin{cases} 0, 0 < x < L \\ V_0, x < 0 \text{ and } x > L \end{cases} \qquad (\text{here, } V_0 > 0)$$

Show that the bound state energies ($E < V_0$) are given by equation 7

$$\tan kL = \frac{2kk'}{k^2 - k'^2}$$

where, $k = \sqrt{\frac{2mE}{\hbar^2}}$ and $k' = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$

(*b*) Use the Schrödinger equation to obtain the expressions for the reflection and transmission co-efficients of a particle of mass m and energy E, approaching a potential step of height V_0 for the case $E < V_0$. 8