

(c) A plane polarized light is incident perpendicularly on a Quarter Wave Plate. Find its thickness which introduces a phase difference of 60° between e - and o -rays. (Given $\mu_e = 1.553$, $\mu_o = 1.544$, $\lambda = 5400 \times 10^{-10}$ m.)

(d) Derive Maxwell's divergence equations from Maxwell's curl equations.

(e) In a homogeneous region, where $\mu_r = 1$ and $\epsilon_r = 50$,

$$\vec{E} = 20\pi e^{j(\omega t - \beta z)} \hat{a}_x \text{ Volts/m, } \vec{H} = H_0 e^{j(\omega t - \beta z)} \hat{a}_y \text{ Tesla.}$$

Here \hat{a}_x , \hat{a}_y are unit vectors in the x and y directions. Find ω and H_0 if the wavelength is 1.78 m.

(f) Starting from the boundary condition satisfied by the electromagnetic fields at an interface between two dielectric media, deduce the Snell's law.

(g) For an optical fibre with refractive index of the core 1.47 and of its cladding 1.46, calculate the pulse dispersion per km.

2. (a) Define scalar and vector potentials. Show that the Maxwell's equations can be expressed as two coupled second order differential equations in terms of scalar and vector potentials. How do the above equations get simplified using Lorentz condition? Discuss the significance of Gauge Transformations.

(b) Show that the time averaged Poynting vector for electromagnetic time-varying fields is given by :

$$\langle \vec{S} \rangle = \frac{1}{2} \text{Re} \left(\vec{E} \times \vec{H}^* \right)$$

where \vec{H}^* is the complex conjugate of the vector \vec{H} .

3. (a) A plane electromagnetic wave propagating in a conducting medium is characterized by the parameters ϵ , μ and σ . Show that propagation constant is complex in this case and is given by :

$$\beta = \omega \sqrt{\epsilon \mu (1 + i\sigma / (\omega \epsilon))}.$$

Hence discuss the propagation of electromagnetic waves in a good and a bad conductor. Here ϵ , μ and σ are the permittivity, permeability and conductivity of the medium.

- (b) Calculate the skin depth for a conductor at 1 GHz, given that $\sigma = 3.8 \times 10^7$ mho/m, $\mu = 2.57 \times 10^{-7}$ H/m. 12,3
4. (a) Derive Fresnel's relations for reflection and refraction of plane electromagnetic waves at an interface between two dielectric media when the electric field vector of the incident wave is parallel to the plane of incidence.
- (b) Discuss the phenomenon of total internal reflection on the basis of electromagnetic theory. Determine the change of phase in the reflected ray when it suffers a total internal reflection. 7,8
5. (a) Show that in an electrically anisotropic dielectric medium the permittivity tensor is symmetric.
- (b) Show that in an anisotropic dielectric medium the electric field, the magnetic field and the Poynting vector on one hand and the electric displacement, the magnetic field and the wave normal on the other hand form orthogonal triplets.
- (c) On putting a polarimeter tube 25 cm long containing a sugar solution of unknown strength, the plane of polarization is rotated through 10 degrees. Given the specific rotation of sugar is 60° per decimetre/(gm/cc), find the concentration of the sugar solution. 4,8,3

6. (a) Derive Fresnel's laws of phase velocities in an electrically anisotropic medium. Show that it leads to the phenomenon of double refraction.
- (b) Describe the construction of Babinet Compensator and explain how it is used to determine the direction of major and minor axes. How is this compensator used to determine the ratio of these axes for an elliptically polarized light ? 7,8
7. (a) Starting with the Maxwell's equations obtain the wave equations for the propagation of an electromagnetic wave in a symmetric planar wave guide. Derive the appropriate eigenvalue equations and show that there exists only one Symmetric TE mode for $0 < V < \pi$, 'V' being the dimensionless wave guide parameter.
- (b) Obtain an expression for the Numerical Aperture of an optical fibre in terms of the refractive indices of the core and cladding. 11,4

Value of constants :

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\epsilon_0 = 9.0 \times 10^{-12} \text{ F/m}$$

$$q_e = 1.6 \times 10^{-19} \text{ C}$$

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$c = 3 \times 10^8 \text{ m/s.}$$