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Your Roll No. ....

1414

B.Sc. (Hons.)/I **A**

STATISTICS—Paper II

(Mathematics—II)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *four* questions in *all*,

selecting *two* questions from each Section.

SECTION I

1. (*a*) Evaluate any *two* of the following :

(*i*) $$\int \frac{x^2+1}{(x+2)^3\,(x-1)}\,dx$$

(*ii*) $$\int \frac{x^3-x^2-1}{(x^2+1)^2\,(x^2-1)}\,dx$$

(*iii*) $$\int_0^\infty \frac{x(\tan^{-1}x)^2}{(1+x^2)^{3/2}}\,dx.$$

(*b*) Show that :

$$\int_1^\infty \frac{x^2+3}{x^6(x^2+1)}\,dx = \frac{1}{30}(58-15\pi). \qquad 6, 3\tfrac{1}{2}$$

2. (*a*) Obtain the reduction formula for

$$\int \sin^p x \cos^q x \; dx.$$

Hence evaluate :

$$\int \frac{\sin^4 x}{\cos^5 x}\,dx.$$

(*b*) Find the limit, when *n* tends to infinity of the sum :

$$\sum_{r=1}^{n} \frac{n^3}{(n^2+r^2)(n^2+2r^2)}. \qquad 5, 4\tfrac{1}{2}$$

3. (*a*) Show that the area of a loop of the curve

$r\cos\theta = a\cos 2\theta$ is $a^2(4-\pi)/2$.

(*b*) Prove that the volume of the solid generated by the revolution of the curve $y = a^3/(a^2+x^2)$ about its asymptote is $\pi^2 a^3/2$. 5, 4½

SECTION II

4. (*a*) If the lines $ax^2 + 2hxy + by^2 = 0$ are the sides of a parallelogram and the line $lx + my = 1$ is one of its diagonals, show that the equation of the other diagonal is :

$$y(bl - hm) = x(am - hl).$$

(*b*) Show that the straight lines given by the equation $(ax + by)^2 - 3(bx - ay)^2 = 0$ form with the line $ax + by + c = 0$ an equilateral triangle whose area is :

$$c^2/\left[\sqrt{3}(a^2 + b^2)\right].$$

5, 4½

5. (*a*) The distances from the origin to the centres of three circles $x^2 + y^2 - 2\lambda x = c^2$ (where c is a constant and λ is a variable) are in G.P. Prove that the lengths of the tangents drawn to them from any point on the circle $x^2 + y^2 = c^2$ are also in G.P.

(*b*) Show that the tangent at any point P on a parabola bisects the angle between the focal chord through P and the perpendicular from P on the directrix. 5, 4½

6. (*a*) Show that the tangent and normal at only point of an ellipse bisect the angles between the focal radii to that point.

(*b*) From a point on the circle $x^2 + y^2 = a^2$, tangents are drawn to the hyperbola $x^2 - y^2 = a^2$. Show that the locus of the middle point of chord of contact is :

$$(x^2 - y^2)^2 = a^2(x^2 + y^2).$$ 5, 4½