This question paper contains 4+2 printed pages]

Your Roll No...................................

1417

B.Sc. (Hons.)/I A

STATISTICS—Paper V

(Statistical Methods—I)

(For Admissions of 1999 and onwards)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Five* questions in all. Q. No. 1 is compulsory. Attempt *Four* more questions, selecting at least *one* question from each Section.

1. (*a*) The mean and median of 100 items are 50 and 52 respectively. The value of the largest item is 100. It was later found that it is actually 110. Therefore the true mean is .... and the true median is ....

P.T.O.

(*b*) $|x - 6| + |x + 3| + |x - 8| + |x + 4| + |x - 3|$ is least when $x =$ ....

(*c*) The mode of the binomial distribution with $n = 7$ and $p = \frac{1}{2}$ is ....

(*d*) In a Poisson distribution, the second moment about the origin is 12, then its third moment about mean is 2,1,1,2

Section I

2. (*a*) What do you understand by skewness ? How is it measured ? Distinguish clearly, by giving figures, between positive and negative skewness. Also show the relative positions of mean, median and mode in the figures, for positively and negatively skewed distributions.

(*b*) Find the mean deviation from the mean and s.d. of A.P. $a, a + d, a + 2d, \ldots\ldots, a + 2nd$ and verify that the latter is greater than the former. 5,3

3. (*a*) Show that in a discrete series if deviations are small compared with mean M so that $\left(\frac{x}{M}\right)^3$ and higher powers of $\left(\frac{x}{M}\right)$ are neglected, we have :

(*i*) $G = M\left(1 - \frac{1}{2}\frac{\sigma^2}{M^2}\right)$

(*ii*) $MH = G^2$

(*iii*) $V = \sqrt{\frac{2(M - G)}{M}}$

where M is the A.M. σ^2 is the variance, G is the G.M. and V is the coefficient of variation.

(*b*) Let r be the range and :

$$S = \left[\frac{1}{n-1}\sum_{i=1}^{n}\left(x_i - \bar{x}\right)^2\right]^{1/2}$$

be the standard deviation of a set of observations $x_1, x_2, \ldots\ldots\ldots, x_n$, then prove that :

$$S \leq r\left(\frac{n}{n-1}\right)^{1/2}.$$ 6,2

Section II

4. (*a*) Show that the best fitting linear function for the points $(x_1, y_1), (x_2, y_2), \ldots\ldots\ldots, (x_n, y_n)$ may be expressed in the form :

$$\begin{vmatrix} x & y & 1 \\ \sum x_i & \sum y_i & n \\ \sum x_i^2 & \sum x_i y_i & \sum x_i \end{vmatrix} = 0 \; (i = 1, 2, \ldots\ldots, n)$$

Show that the line passes through the mean point $(\bar{x}, \bar{y})$.

(*b*) X and Y are two r.v's with variances σ_X^2 and σ_Y^2 respectively and r is the coefficient of correlation between them. If $U = X + kY$ and $V = X + (\sigma_X / \sigma_Y) Y$, find the value of k so that U and V are uncorrelated. 4,4

5. (*a*) Define :

(*i*) lines of regression and

(*ii*) regression coefficients.

Show that the coefficient of correlation is the geometric mean between the regression coefficients.

(*b*) Show that :

$$1 - R_{1.23}^2 = \left(1 - r_{12}^2\right)\left(1 - r_{13.2}^2\right).$$

Deduce that if $R_{1.23} = 0$, X_1 is uncorrelated with any of other variables. 4,4

Section III

6. (*a*) Obtain the moment generating function of the binomial distribution. Derive from it the result that the sum of two binomial variates is a binomial variate if the variates are independent and have the same probability of success.

(*b*) If X and Y are independent Poisson variates with means m_1 and m_2 respectively, prove that the probability that $X - Y$ has the value r is the coefficient of t^r in :

$$\exp\left(m_1 t + m_2 t^{-1} - m_1 - m_2\right).$$ 4,4

7. (*a*) If X is a negative binomial variate with p.m.f. :

$$f(x) = \begin{cases} \binom{k+x-1}{x} q^x \, p^k; & x = 0, 1, 2, \ldots\ldots\ldots \\ 0 & \text{otherwise} \end{cases}$$

then show that the moment recurrence formula is :

$$\mu_{r+1} = q\left(\frac{d\mu_r}{dq} + \frac{rk}{p^2}\,\mu_{r-1}\right).$$

(*b*) Obtain *r*th factorial moment of hypergeometric distribution. Hence obtain the values of mean and variance. 4,4