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Your Roll No.....

1413

B.Sc. (Hons.)/I **A**

STATISTICS—Paper I

(Mathematics–I)

(Admissions of 1999 and onwards)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Answer *four* questions in all,

selecting *two* questions from each Section.

SECTION I

1. (*a*) Show that the function f defined by

$$f(x) = |x - 2| + |x| + |x + 2|$$

is not derivable at $x = -2$, 0 and 2.

(*b*) If $x = \sin t$, $y = \sin pt$, prove that :

$$(1 - x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + p^2y = 0.$$

P.T.O.

(*c*) If $y^{1/m} + y^{-1/m} = 2x$, prove that :

$$(x^2 - 1)y_{n+2} + (2n + 1)xy_{n+1} + (n^2 - m^2)y_n = 0. \quad 3,3,3\frac{1}{2}$$

2. (*a*) A rectangular sheet of metal has four equal square portions removed at the corners and the sides are then turned up so as to form an open rectangular box. Show that, when volume contained in the box is a maximum, the depth will be :

$$\frac{1}{6}[(a + b) - (a^2 - ab + b^2)^{1/2}]$$

where a, b are the sides of the original rectangle.

(*b*) If

$$u = \tan^{-1} \frac{x^3 + y^3}{x - y}, \quad x \pm y,$$

show that :

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u) \sin 2u.$$

(*c*) Show that the points of inflexion of the curve :

$$y^2 = (x - a)^2 (x - b)$$

lie on the line $3x + a = b$. 3,3,3½

3. (*a*) Define an asymptote. Find the asymptotes of the curve :

$$x^2y^2\ (x^2 - y^2)^2 = (x^2 + y^2)^3.$$

(*b*) Determine the position and nature of the double points on the curve

$$y^2\ (a^2 + x^2) = x^2(a^2 - x^2).$$ 5,4½

SECTION II

4. (*a*) Trace the curve : 2

$$y^2\ (x^2 - 1) = 2x - 1.$$

(*b*) Solve the following differential equations :

(*i*) $x\sqrt{y}\,dx + (1 + y)\,\sqrt{1 + x}\,dy = 0.$

(*ii*) $\dfrac{d^2y}{dx^2} + 4\dfrac{dy}{dx} + 3y = e^{-3x}.$ 4½,5

5. (*a*) Trace the curve :

$$x = a(\theta + \sin\theta),\ y = a(1 + \cos\theta),$$

$$-\pi \leq \theta \leq \pi$$

P.T.O.

(*b*) Solve the following equations :

(*i*) $x \frac{dy}{dx} = y - \sqrt{x^2 + y^2}$.

(*ii*) $p^2 + 2py \cot x = y^2$; $p = \frac{dy}{dx}$. 4½, 5

6. Solve any *four* of the following :

(*i*) $x^2 \frac{d^2y}{dx^2} + 7x \frac{dy}{dx} + 5y = 2x^6$.

(*ii*) $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = \cos 2x$.

(*iii*) $\sqrt{x^2 + 1} \frac{dy}{dx} + y = \sqrt{x^2 + 1} - x$.

(*iv*) $(D^2 + 4)y = \sin 2x$.

(*v*) $(D^2 - 2D + 4)y = e^x \cos x$.

(*vi*) $\frac{dy}{dx} + \frac{y}{x-1} = ny^{1/3}$ 9½