[This question paper contains 4 printed pages.]

1415 *Your Roll No.* ....

B.Sc. (Hons.) / I **A**

STATISTICS – Paper III

(Mathematics – III)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Four** *questions in all, selecting at least* **two** *questions from each Section.*

SECTION I

1. (a) If $\cos\alpha + \cos\beta + \cos\gamma = 0$ and $\sin\alpha + \sin\beta + \sin\gamma = 0$ then prove that

$$\sum \cos 4\alpha = 2\sum \cos 2(\beta + \gamma)$$

$$\sum \sin 4\alpha = 2\sum \sin 2(\beta + \gamma).$$

(b) Prove that if n is a positive integer, then

$$\cos\theta + {}^{n}c_{1}\cos(\theta + \varphi) + {}^{n}c_{2}\cos(\theta + 2\varphi) + \cdots + {}^{n}c_{n}\cos(\theta + n\varphi) = \left(2\cos\frac{\varphi}{2}\right)^{n}\cos\left(\theta + \frac{n\varphi}{2}\right)$$

P.T.O.

(c) Using De'Moivre's theorem, express $\tan 8\theta$ in terms of powers of $\tan\theta$. (3,4½,2)

2. (a) Solve the equation

$$x^4 - 8x^3 + 21x^2 - 20x + 5 = 0$$

the sum of two of the roots being equal to the sum of the other two.

(b) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$, find the values of

(i) $\sum\alpha^2\beta^2$ (ii) $\sum\alpha^3\beta$ (iii) $\sum\frac{1}{\beta+\gamma}$ (5,4½)

3. (a) Prove that if n is a positive integer, then

(i) $\left(1-\frac{1}{n}\right)^n < \left(1-\frac{1}{n+1}\right)^{n+1}$

(ii) $\left(1+\frac{1}{n}\right)^{n+1} > \left(1+\frac{1}{n+1}\right)^{n+2}$

(b) If $(a_1, a_2, \cdots a_n)$ and $(b_1, b_2, \cdots b_n)$ are any real number (positive, zero or negative), then show that

$$(a_1b_1 + a_2b_2 + \cdots + a_nb_n)^2 \le (a_1^2+a_2^2 + \cdots + a_n^2)(b_1^2 + b_2^2 + \cdots + b_n^2)$$

and equality occurs iff $\frac{a_1}{b_1} = \frac{a_2}{b_2} = \cdots = \frac{a_n}{b_n}$

Hence or otherwise show that for $a_i > 0$ and $b_i > 0$, ($i = 1, 2, --- n$)

$$\left(\frac{a_1}{b_1} + \frac{a_2}{b_2} + --- + \frac{a_n}{b_n}\right)(a_1b_1 + a_2b_2 + --- + a_nb_n) \geq (a_1^2 + a_2^2 + --- + a_n^2).$$

(4,5½)

SECTION II

4. (a) Prove that the transpose of the product of two matrices is equal to the product of transposes taken in reverse order.

(b) If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then show that

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$ for every positive integer n.

(c) Give examples of 3×3 Hermitian and Skew-Hermitian matrices. Show that every square matrix can be uniquely expressed as P + i Q where P and Q are Hermitian matrices. (3,3,3½)

5. (a) Prove that

$$\begin{vmatrix} o & -c & b & -l \\ c & o & -a & -m \\ -b & a & o & -n \\ x & y & z & o \end{vmatrix} = (al + bm + cn)(ax + by + cz)$$

(b) Express $\begin{vmatrix} -a^2 & ab & ac \\ ab & -b^2 & bc \\ ac & bc & -c^2 \end{vmatrix}$ as a square of a determinant and hence evaluate it.

(c) Prove that a skew symmetric determinant of odd order vanishes. (4,3½,2)

6. (a) If A is a symmetric matrix, then prove that the adjoint of A is also symmetric. Also, show that the inverse of a matrix, if it exists, is unique.

(b) Find the value of $\text{adj}(p^{-1})$ in terms of P where P is a non-singular matrix and hence show that $\text{adj}(Q^{-1} B P^{-1}) = PAQ$ given that $\text{adj } B = A$ and $|P| = |Q| = 1$. (4½,5)