This question paper contains 4+1 printed pages]

Your Roll No.....................................

1416

B.Sc. (Hons.)/I **A**

STATISTICS—Paper IV

(Probability Theory—I)

(For admission of 1999 and onwards)

Time : 2 *Hours* *Maximum Marks* : 38

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Four* questions in all,

selecting *two* questions from each Section.

Section I

1. (*a*) Give the classical and statistical definitions of probability. What are the limitations in respect of these definitions ?

(*b*) One shot is fired from each of the three guns E_1, E_2, E_3 denote the events that the target is hit by the first, second and third guns respectively. If $P(E_1) = 0.5$, $P(E_2) = 0.6$ and

$P(E_3) = 0.8$ and E_1, E_2, E_3 are independent events, find the probability that :

(*i*) exactly one hit is registered and

(*ii*) at least two hits are registered.

(*c*) Show that :

$$P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B).$$ 3,3½,3

2. (*a*) A man is equally likely to choose one of three routes A, B, C from his house to the railway station, and his choice of route is not influenced by the weather. If the weather is dry, the probabilities of missing the train by routes A, B, C are respectively 1/20, 1/10, 1/5. What is the probability that the route chosen was C ?

(*b*) Define a random variable 'X' and cumulative distribution function $F_X(x)$. Show that $F_X(x)$ is monotonic non-decreasing function of x and $0 \leq F_X(x) \leq 1 \ \forall \ x \in R$.

(c) If X and Y are independent r.v.'s such that :

$$f(x) = e^{-x},\ x \geq 0;\ g(y) = 3e^{-3y};\ y \geq 0,$$

find the probability distribution of $Z = \frac{X}{Y}$. 3,3,3½

3. (a) The distribution function of a r.v. X is given by :

$$F(x) = \begin{cases} 1-(1+x)\,e^{-x} & ; \quad \text{for } x \geq 0 \\ 0 & ; \quad \text{for } x < 0 \end{cases}$$

Find :

(i) pmf/pdf of r.v. X

(ii) P(X = 0), P(X ≥ 2) and P(2 ≤ X ≤ 6).

(b) X is a r.v. with mean zero and cumulants k_r, $r = 1, 2, 3, 4$. Find the first two cumulants l_1 and l_2 of X^2 in terms of k_i, $i = 1, 2, 3, 4$.

(c) State and prove addition theorem of expectation. What is the expectation of the number of failures preceeding the first success in an infinite series of independent trials with constant probability 'p' of success in a trial ? 4,2,3½

Section II

4. (*a*) Define log-normal distribution. Find its rth moment about origin. Hence find its mean and variance.

(*b*) If $X \sim N(\mu, \sigma^2)$, obtain the pdf of :

$$U = \frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2$$

and identify its distribution.

(*c*) Show that the exponential distribution "Lacks Memory".

3½,3,3

5. (*a*) If the joint distribution function of X and Y is given by :

$$F(x, y) = \begin{cases} \left(1 - e^{-x}\right)\left(1 - e^{-y}\right) & , \quad x > 0, y > 0 \\ 0 & , \quad \text{elsewhere} \end{cases}$$

Find :

(*i*) marginal distribution function of X

(*ii*) $P(X \leq 2)$, $P(X \leq 3, Y \leq 5)$ and $P(Y > 2)$

(*iii*) joint pdf $f(x, y)$ of (X, Y) and

comment on the independence of X, Y.

(*b*) Find the mode of a beta distribution of second kind.

6,3½

6. (*a*) Show that, for rectangular distribution :

$$f(x) = \frac{1}{2a}, -a < x < a,$$

m.g.f. about origin is :

$$\frac{1}{at}(\sinh at).$$

Also, show that moments of even order are given by :

$$\mu_{2n} = \frac{a^{2n}}{(2n+1)}, n = 0, 1, 2 \ldots\ldots\ldots .$$

(*b*) Let X and Y be independent standard normal variates. Find the distribution of $\frac{X}{Y}$ and identify it.

(*c*) Find cumulant generating function of gamma distribution. Hence, find its mean and variance. 3½,3,3