[This question paper contains 4 printed pages.]

1041   *Your Roll No.* ....

*(Write your Roll No. on the top immediately
on receipt of this question paper.)*

Answer **four** *questions in all,
selecting* **two** *questions from each Section.*

SECTION I

1. (a) Examine the continuity and derivability of the
    function :

$$f(x) = \begin{cases} \dfrac{xe^{\frac{1}{x}}}{1+e^{\frac{1}{x}}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$$

    at $x = 0$.

    (b) Find the nth derivative of

$$y = \frac{x}{x^2 + a^2}$$

                                                        P.T.O.

(c) If $y = e^{m \sin^{-1} x}$ show that :

$$(1 - x^2)y_{n-2} - (2n + 1)xy_{n-1} = (n^2 + m^2)y_n = 0$$

and hence evaluate $(y_n)_0$.　　　　　　(3,2½,4)

2.　(a) Show that the semi vertical angle of the cone of maximum volume and of given slant height is $\tan^{-1} \sqrt{2}$.

(b) If $v = r^m$, where $r^2 = x^2 + y^2 + z^2$, show that :

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} = m(m + 1)r^{m-2}$$

(c) Find the points of inflexion on the curve

$$(x^2 + a^2)y = a^2 x.$$ 　　　　　　(3,3,3½)

3.　(a) Find all the asymptotes of the curve :

$$(x + y)^2(x + 2y + 2) = x + 9y + 2.$$

(b) Find the position and nature of the double points on the curve :

$$y^2(a^2 + x^2) = x^2(a^2 - x^2).$$ 　　　　(5,4½)

SECTION II

4.   (a) Trace the curve :

$$ay^2 = x^2(a - x).$$

   (b) Solve the following differential equations :

      (i) $x\sqrt{y}\,dx + (1 + y)\sqrt{1 + x}\ dy = 0$

      (ii) $\sin x \dfrac{dy}{dx} + y \cos x = x \sin x$ (4½,5)

5.   (a) Trace the curve

     $x = a(\theta + \sin\theta),\ y = a(1 + \cos\theta),\ -\pi < \theta < \pi$

   (b) Solve the following equations :

      (i) $\dfrac{dy}{dx} = (4x + y + 1)^2$

      (ii) $y = 2px + y^2p^3,\ p \equiv \dfrac{dy}{dx}$ (4½,5)

6.   Solve any **four** of the following :

      (i) $x(y^2 - x^2 - a^2x)dx + y(y^2 + x^2 - b^2y)dy = 0$

      (ii) $x^2\dfrac{d^2y}{dx^2} + 7x\dfrac{dy}{dx} + 5y = 2x^6$

(iii) $(D - a)^2 y = 0$

(iv) $(D^2 + 4D - 3)y = 2 \sin 3x$

(v) $\dfrac{d^2 y}{dx^2} - 2\dfrac{dy}{dx} + 4y = e^x \cos x$

(vi) $\dfrac{dy}{dx} + \dfrac{y}{x-1} = x y^{1/3}$ (9½)