[This question paper contains 4 printed pages.]

1042 *Your Roll No.* .....

B.Sc. (Hons.) / I **C**

STATISTICS – Paper II

A–222 : (Mathematics – II)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **four** *questions in all, selecting* **two** *questions from each Section.*

SECTION A

1. Attempt any **three** of the following integrals :

(a) $\int \frac{x^2 e^{m \sin^{-1} x}}{\sqrt{1-x^2}} dx$

(b) $\int \frac{x}{x^3 + x^2 + x + 1} dx$

(c) $\int_0^{\pi/2} \log \sin x dx$

P.T.O.

(d) $\int \frac{dx}{(x+1)\sqrt{x^2-1}}$

(e) $\int \frac{xdx}{\sqrt{8+x-x^2}}$ (3,3,3½)

2. (a) Show that

$$\lim_{n\to\infty}\left[\frac{n}{n^2+1^2}+\frac{n}{n^2+2^2}+\frac{n}{n^2+3^2}+.....+\frac{1}{2n}\right]=\frac{\pi}{4}$$

(b) If $I_n = \int x^n\sqrt{a-x}\,dx$, prove that

$(2n+3)I_n = 2anI_{n-1} - 2x^n(a-x)^{3/2}$.

Hence evaluate $\int_0^a x^2\sqrt{ax-x^2}\,dx$. (4½,5)

3. (a) Show that the area of the region included between the cardiodes :

$$r = a(1-\cos\theta),\ r = a(1-\cos\theta)$$

is $a^2(3\pi-8)/2$.

(b) Find the volume of the solid obtained by revolving the loop of the curve :

$$y^2(a+x) = x^2(3a-x)$$

about x-axis. 4½,5)

SECTION B

4. (a) If $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ represents two straight lines, prove that the square of the distance of their point of intersection from the origin is

$$\{c(a + b) - f^2 - g^2\}/(ab - h^2)$$

(b) Show that the equation $ay^2 - 2hxy + bx^2 = 0$ represents the pair of straight lines through the origin perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$. (4½,5)

5. (a) Prove that a normal chord of a parabola which subtends a right angle at the vertex makes an angle $\tan^{-1}\sqrt{2}$ with the axis.

(b) Find the equation to the pair of straight lines obtained by joining the origin to the points of intersection of the straight line $y = mx + c$ and the circle $x^2 + y^2 = a^2$ and prove that they are at right angles if $2c^2 = a^2(1 + m^2)$. (4½,5)

6. (a) If two points are taken on the minor axis of an ellipse at the same distance from the centre as

the foci, prove that the sum of the squares of the perpendicular from these points on any tangent to the ellipse is constant.

(b) Show that the locus of the middle points of normal chords of the rectangular hyperbola

$x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$. (4½,5)