[This question paper contains 4 printed pages.]

1043 *Your Roll No.* ....

B.Sc. (Hons.) / I **C**

STATISTICS – Paper III

A–223 : (Mathematics – III)

(Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Four** *questions in all, selecting at least* **two** *questions from each Section.*

SECTION I

1. (a) Find a necessary condition for the sum of two roots of the equation

$$x^4 - px^3 + qx^2 - rx + s = 0$$

to be equal to the sum of the other two.

(b) If α, β, γ be the roots of the equation

$$x^3 - px^2 + qx - r = 0,$$

find the value of

(i) $(\beta + \gamma)(\gamma + \alpha)(\alpha + \beta)$ (ii) $\sum\left(\frac{\alpha}{\beta}\right)$.

P.T.O.

(c) Form the cubic whose roots are the values of α, β, γ given by the relations

$$\sum\alpha = 3,\ \sum\alpha^2 = 5,\ \sum\alpha^3 = 11$$

Hence find the value of $\sum\alpha^4$. (2,3,4½)

2. (a) If $a = \cos\alpha + i\sin\alpha$, $b = \cos\beta + i\sin\beta$, $c = \cos\gamma + i\sin\gamma$.

Prove that $\frac{(b+c)(c+a)(a+b)}{abc}$ is real and equal to

$$8\cos\frac{1}{2}(\beta-\gamma)\cos\frac{1}{2}(\gamma-\alpha)\cos\frac{1}{2}(\alpha-\beta).$$

(b) Sum the series $\cos\theta + x\cos 2\theta + x^2\cos 3\theta + \ldots$ to n terms. (4,5½)

3. (a) If a, b, c be all positive, then show that

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{3}{2}.$$

(b) State and prove Cauchy-Schwartz inequality for the two sets of real numbers $(a_1, a_2, \ldots a_n)$ and $(b_1, b_2, \ldots b_n)$. Hence or otherwise, for the real numbers $(c_1, c_2, \ldots c_n)$, prove that

$$(c_1 + c_2 + \ldots c_3)^2 \leq n^2 (c_1^3 + c_2^3 + \ldots c_n^3)$$ (4,5½)

SECTION – II

4. (a) If $A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, Prove that $(aI + bA)^n = a^n I + n\, a^{n-1}\, b\, A$

Where I is 2×2 rowed unit matrix.

(b) If A and B are idempotent matrices, then (A+B) will be idempotent if and only if AB=BA=0.

(c) Prove that every square matrix can be uniquely expressed as the Sum of a Symmetric and Skew Symmetric matrix. (2,4,3½)

5. (a) Solve the following system of linear equations by CRAMER'S rule

$x + ay + a^2 z + a^3 = 0,$

$x + by + b^2 z + b^3 = 0,$

$x + cy + c^2 z + c^3 = 0.$

(b) Express $\Delta = \begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix}$

as a product of two determinants and find its value. (4½,5)

6. (a) If $A = I_n - X(X'X)^{-1}X'$; verify the following

(i) Is A Square matrix

(ii) Is A Idempotent matrix

(iii) Is A Symmetric matrix, if X is non-singular

(b) Let e be the $(n \times 1)$ column vector with elements $(1,1......1)$, A be $(n \times n)$ matrix and I_n be the unit matrix. Let $M(x) = I_n - x\,e\,e'$ where x is a scalar

(i) Prove that $M(x)M(y) = M(x + y + kxy)$ where k is the scalar $e'Ae$

(ii) Verify that reciprocal of $M(x)$ is $M\left(\frac{-x}{1 + kx}\right)$.

(5,4½)