[This question paper contains 4 printed pages.]

1044 *Your Roll No.* ....

B.Sc. (Hons.) / I **C**

STATISTICS – Paper IV

A-224 : (Probability Theory – I)

(For Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Four** *questions in all, selecting* **two** *questions from each Section.*

SECTION I

1. (a) State and prove addition theorem of probability for two events. Hence show

   (i) $P(A \cup B) \leq P(A) + P(B)$

   (ii) $P(A \cap B) \geq P(A) + P(B) - 1$

   (b) A factory needs two raw materials, say X and Y. The probability of not having an adequate supply of material X is 0.06, whereas the probability of not having an adequate supply of material Y is

0.04. A study shows that the probability of a shortage of both X and Y is 0.02. What is the probability of the factory being short of either material X or Y ?

(c) Prove that for n arbitrary independent events $A_1, A_2, \ldots, A_n$,

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots \cup A_n) + P(\bar{A}_1)\, P(\bar{A}_2)\, P(\bar{A}_3) \ldots P(\bar{A}_n) = 1.$$

(3,3,3½)

2. (a) The police plans to enforce speed limits by using radar traps at 4 different locations within the city limits. The radar traps at each of the locations L_1, L_2, L_3 and L_4 are operated 40%, 30%, 20%, and 30% of the time. If a person who is speeding on his way to work has probabilities of 0.2, 0.1, 0.5 and 0.2 respectively, of passing through these locations, what is the probability that he will receive a speeding ticket ?

(b) The distribution function of a r.v. X is given by :

$$f(x) = \begin{cases} 1-(1+x)e^{-x}; & \text{for } x \ge 0 \\ 0 & : \text{for } x < 0 \end{cases}$$

Find :

(i) pmf/pdf of r.v. X.

(ii) $P(X = 3)$, $P(X \le 5)$ and $P(-2 \le X \le 3)$.

(c) Suppose that X has pdf :

$$f(x) = 2x, \ 0 < x < 1.$$

Find the pdf of $Y = X^3 - 1$. (3½,4,2)

3. (a) Let the joint pmf of X_1 and X_2 be

$$p(x_1, x_2) = (x_1 + x_2)/21; \ x_1 = 1, 2, 3; \ x_2 = 1, 2.$$

Find

(i) marginal pmf of X_1.

(ii) conditional pmf of X_1 given $X_2 = 1$.

Examine the independence of X_1 and X_2.

(b) State and prove addition theorem of expectation.

(c) A box contains 2^n tickets among which ${}^nc_i$ tickets bear the number i; i = 0, 1, 2, ..., n. A group of m tickets is drawn. What is the expectation of the sum of their numbers ? (4,2½,3)

SECTION II

4. (a) If X has a uniform distribution in [0,1], find the p.d.f. of $Y = -2 \log X$. Identify the distribution also.

(b) Let X be a random variable following normal distribution with mean μ and variance σ^2. Show that

$$\mu'_{r+2} = 2\mu\mu'_{r+1} + (\sigma^2 - \mu^2)\mu_r' + \sigma^3 \, d\mu_r'/d\sigma;$$ where r is non-negative integer.

P.T.O.

(c) Find cumulant generating function of gamma distribution. Hence, find its mean and variance. (3,4,2½)

5. (a) If X has an exponential distribution parameter θ then, show that for every constant $a \geq 0$,

$$P(X - a \leq x \quad X \geq a) = P(X \leq x); \text{ for all } x \geq 0.$$

Also name this property.

(b) Find mean deviation about mean for standard laplace distribution.

(c) If X is a variable with zero mean and cumulants k_r, r = 1, 2, 3, ... find the first two cumulants of X^2. (3½,3,3)

6. (a) Let $X \sim N(0, 1)$ and $Y \sim N(0, 1)$ be independent random variables. Find the distribution of X/Y and identify it.

(b) The joint pdf of the two-dimensional random variable (X, Y) is of the form :

$f(x,y) = ke^{[illegible]}$, $0 \leq y < x < \infty$.

(i) Determine the constant k.

(ii) Find marginal pdf of X.

(iii) Find conditional pdf of Y given X = 2.

(iv) Examine if X, Y are independent. (4½,5)