[This question paper contains 6 printed pages.]

1045 *Your Roll No.* ....

B.Sc. (Hons.) / I **C**

STATISTICS – Paper V

A–225 (Statistical Methods – I)

(For Admissions of 1999 and onwards)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt **Five** *questions in all. Q. No.* **1** *is compulsory. Attempt* **four** *more questions, selecting at least* **one** *question from each Section.*

1. (a) The mean of 30 observations is 16. On checking it was found that two observations were wrongly copied as 8 and 4. If wrong observations are replaced by correct values 6 and 9, then the correct mean is ____ .

(b) Obtain Pearson's coefficient of skewness, when mean, mode and standard deviations are 31, 36 and 6 respectively.

(c) The mode of binomial distribution with n = 8 and p = 1/3 is ____ .

(d) The equations of two regression lines obtained in a correlation analysis are as follows :

$3X + 12Y - 19 = 0$ and $3Y + 9X = 46$. Obtain :

(i) The value of correlation coefficient.

(ii) The ratio of coefficient of variability of X to that of Y.

(e) If all A's are B's and all B's are C's, then show that all A's are C's. (1,1,1,2,1)

SECTION I

2. (a) If $\bar{x}_w$ is the weighted mean of x_i's with weights w_i, $i = 1, 2, \ldots, n$, prove that :

$$\sum_{i=1}^{n} w_i \left[\sum_{i=1}^{n} w_i \left(x_i - \bar{x}_w\right)^2\right] = \sum_{i=1}^{n} \sum_{j>i}^{n} w_i w_j \left(x_i - x_j\right)^2,$$

where $\sum_{i=1}^{n} w_i \neq 0$.

(b) From a sample of n observations, the arithmetic mean and variance are calculated. It is then found that one of the values, x_1, is in error and should be replaced by x_1'. Show that the adjustment to the variance to the correct this error is :

$$\frac{1}{n}\left(x_1' - x_1\right)\left(x_1' + x_1 - \frac{x_1' - x_1 + 2T}{n}\right),$$

where T is the total of the original results.

(c) Show that in a discrete series if deviations $(X_i - M)$ are small compared with mean M, then

$$G = M\left(1 - \frac{\sigma^2}{2M^2}\right),$$

where M is the arithmetic mean, G the geometric mean and σ is the standard deviation of the distribution. (3,3,2)

3. (a) Show that, for a random variable X, if moments of all order exist, then $\mu_{2j+1}^2 \le \mu_2, \mu_{2j+2}$, where μ_j is the j^{th} central moment.

(b) Show that for a discrete distribution, $\beta_2 \ge 1$.

(c) Define 'moments'. Express r^{th} moment about mean (μ_r) in terms of various moments about an arbitrary point 'a' (μ_r'). Hence express μ_4 in terms of μ_1', μ_2', μ_3' and μ_4'. (3,2,3)

SECTION II

4. (a) What is 'Legender's principle of least squares' ? Derive the normal equations for fitting the curve $Y = aX + (b/X)$ to a set of n points (X_i, Y_i), $i = 1, 2, \ldots, n$.

(b) X_1, X_2 are two variates with variances σ_1^2 and σ_2^2 respectively and ρ is the correlation coefficient between them. Determine the values of the constants 'a' and 'b' which are independent of ρ such that $X_1 + aX_2$ and $X_1 + bX_2$ are uncorrelated.

(c) Show that if X' and Y' are the deviations of the random variables X and Y from their respective means, then

(i) $r = 1 - \frac{1}{2N}\sum\left(\frac{X_i'}{\sigma_X} - \frac{Y_i'}{\sigma_Y}\right)^2$

(ii) $r = -1 + \frac{1}{2N}\sum\left(\frac{X_i'}{\sigma_X} - \frac{Y_i'}{\sigma_Y}\right)^2$ (3,2,3)

5. (a) Define partial and multiple correlation coefficients. In the usual notations, prove that,

$$R^2_{1.23} = \frac{r_{12}^2 + r_{13}^2 - 2r_{12}r_{23}r_{31}}{1 - r_{23}^2} \geq r_{12}^2 .$$

(b) In usual notations, prove that the equation of plane of regression of X_1 on X_2 and X_3 is given by

$$\frac{X_1}{\sigma_1}\omega_{11} + \frac{X_2}{\sigma_2}\omega_{12} + \frac{X_3}{\sigma_3}\omega_{13} = 0 .$$ (4,4)

SECTION III

6. (a) Show that for binomial distribution with parameters n and p,

$$k_{r+1} = pq\frac{dk_r}{dp}.$$

where k_r is the r^{th} cumulant. Hence deduce the values of β_1 and β_2.

(b) If X is a Poisson variate with parameter m and μ_r is the r^{th} central moment, prove that

$$m\left({}^rC_1\mu_{r-1} + {}^rC_2\mu_{r-2} + \dots + {}^rC_r\mu_0\right) = \mu_{r+1}.$$

(c) Show that poisson distribution is a limiting case of negative binomial distribution. (3,3,2)

7. (a) Let X be a discrete random variable having geometric distribution with parameter p. Obtain its mean and variance. Also, show that for any two positive integers s and t.

$$P[X > s + t/X \geq s] = P[X > t].$$

(b) What is a hypergeometric distribution ? Find the mean and variance of this distribution. How is this distribution related to binomial distribution.

(c) Given that $(A) = (\alpha) = (B) = (\beta) = (C) = (\gamma) = \frac{N}{2}$ and also that $(ABC) = (\alpha\beta\gamma)$, show that $2(ABC) = (AB) + (AC) + (BC) - \frac{N}{2}$. (3,3,2)