This question paper contains 4+2 printed pages]

Your Roll No. ....

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B.Sc. (Hons.)/I Sem. **B**

STATISTICS—Paper STH-102

(Calculus—I)

(Admissions of 2011 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all selecting *four* from Section I and *two* from Section II.

SECTION I

1. (*a*) Examine the continuity of the function f defined by :

$$f(x) = \begin{cases} 2x+1 & , \quad 0 < x < \frac{1}{2} \\ 1 & , \quad x = \frac{1}{2} \\ 1-x & , \quad \frac{1}{2} < x < 1 \\ 2x-2 & , \quad x \geq 1 \end{cases}$$

at the points $x = \frac{1}{2}$ and 1. In case of discontinuity, discuss the nature of discontinuity.

P.T.O.

(*b*) Given

$$f(x) = x^2 \sin\left(\frac{1}{x}\right),$$

when $x \neq 0$. Show that f is derivable for every value of x.

(*c*) Show that the expression

$$\left[(x+1)^2/(x+3)^3\right]$$

has a maximum value 2/27 and a minimum value 0.

4½,3½,4½

2. (*a*) If

$$y = x(x+1)\log(x+1)^3,$$

prove that :

$$\frac{d^n y}{dx^n} = \frac{3(-1)^{n-1}(n-3)!(2x+n)}{(x+1)^{n-1}},$$

provided that $n \geq 3$.

(*b*) If

$$y = (1+x^2)^{m/2} \sin(m \tan^{-1} x),$$

show that :

$$y_{2n}(0) = 0$$

and $y_{2n+1}(0) = (-1)^n m (m-1) \ldots\ldots\ldots (m-2n)$.

(*c*) Show that minimum value of :

$u = xy + (a^3/x) + (a^3/y)$ is $3a^2$. 3,7,2½

3. (*a*) If

$$\tan u = \frac{\cos x}{\sinh y} \quad \text{and} \quad \tanh v = \frac{\sin x}{\cosh y}$$

prove that :

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

(*b*) Find dy/dx for the following :

$$(\tan x)^y + y^{\cot x} = a.$$

(*c*) If

$$x = c\cosh\xi\cos\eta, \quad y = c\sinh\xi\sin\eta,$$

show that :

$$\frac{\partial(x, y)}{\partial(\xi, \eta)} = \frac{1}{2}c^2(\cosh 2\xi - \cos 2\eta)$$ 5½,3,4

4. (*a*) Find the points of inflexion of the curve :

$$y = \frac{a^2(a - x)}{a^2 + x^2}$$

(*b*) Trace the curve :

$$y^2 = (x - a)^2 (x - b)$$

for different values of *a* and *b*. 4,8½

5. (*a*) Find the asymptotes of the curve :

$$4x^4 - 13x^2y^2 + 9y^4 + 32x^2y - 42y^3 - 20x^2 + 74y^2 - 56y + 4x + 16 = 0$$

and show that they pass through the intersection of the curve with $y^2 + 4x = 0$.

(*b*) Find the position and nature of the multiple points on the following curve :

$$x^4 + y(y+4a)^3 + 2x^2(y-5a)^2 = 5a^2x^2 .$$ 7½,5

SECTION II

6. (*a*) Solve the initial value problem :

$$(2x\cos y + 3x^2y)\, dx + (x^3 - x^2\sin y - y)\, dy = 0$$

when $y(0) = 2$.

(*b*) Solve :

$$y + xp\log p = (2 + 3\log p)p^3 .$$

(*c*) Solve the following differential equation :

$$\left(\frac{dy}{dx}\right)^3 y^2 - 2x\left(\frac{dy}{dx}\right) + y = 0 .$$ 4,4½,4

7. Solve the following differential equations. Attempt any *four* parts : 12½

(*i*) $y^2\, dx + (3xy - 1)\, dy = 0$

(*ii*) $(2x+y+1)\,dx+(4x+2y-1)\,dy=0$

(*iii*) $\dfrac{dy}{dx}=\dfrac{\sin x+x\cos x}{y\,(2\log y+1)}$

(*iv*) $x^3\dfrac{dy}{dx}-x^2y+y^4\cos x=0$

(*v*) $\dfrac{dy}{dx}+\dfrac{x^2+3y^2}{3x^2+y^2}=0$

8. Solve the following differential equations. Attempt any *three* parts : 12½

(*i*) $(4D^2+16D-9)\,y=4e^{x/2}+3\sin x/4$

(*ii*) $(D^3+D^2+D+1)\,y=x^5-2x^2+x$

(*iii*) $(x^3D^3+3x^2D^2+xD+1)\,y=x\log x$

(*iv*) $\dfrac{d^4y}{dx^4}-y=x^2\sin x$.