This question paper contains 7 printed pages]

Your Roll No................

6034

B.Sc. (Hons.)/I Sem. **B**

STATISTICS—Paper STH–103

(Algebra–I)

(Admissions of 2011 and onwards)

Time : 3 Hours *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *Six* questions in all, selecting *three* questions from each Section.

Section I

1. (*a*) Use the identity

$$a^2(b-c)+b^2(c-a)+c^2(a-b)=-(b-c)(c-a)(a-b)$$

to prove that :

$$\sum \sin\frac{1}{2}(\beta-\gamma)\sin\frac{1}{2}(4\alpha+\beta+\gamma)$$

$$=4\sin\frac{1}{2}(\beta-\gamma)\sin\frac{1}{2}(\gamma-\alpha\sin\frac{1}{2}(\alpha-\beta)\sin(\alpha+\beta+\gamma).$$

P.T.O.

(*b*) If $2^a \sin^2 \theta \cos^6 \theta = b \cos 8\theta + c \cos 6\theta + d \cos 4\theta + e \cos 2\theta + f$, then find the values of a, b, c, d, e and f.

(*c*) Prove that there are four values of θ, lying between 0 and 2π, which satisfy the equation

$$\cos 2\theta + p \cos \theta + q \sin \theta + r = 0$$

and hence show that the sum of four values of θ, which satisfy the equation is an odd multiple of π radians. 5,2,5½

2. (*a*) If α, β, γ are roots of equation $x^3 - x + 2 = 0$, then find an equation whose roots are

$$(\alpha + \beta)^2, (\beta + \gamma)^2 \text{ and } (\gamma + \alpha)^2.$$

(*b*) If α, β, γ are rots of cubic equation $x^3 + px - q = 0$, then find vlue of $\alpha^9 + \beta^9 + \gamma^9$. 6,6½

3. (*a*) Show that

$$\left[\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \ldots\ldots + \frac{1}{\sqrt{n}}\right) \div \sqrt{n}\right] \leq (2n - 1)^{1/4}$$

where n is a positive integer.

(*b*) If α, β, γ are any three positive real numbers such that $\alpha^2 + \beta^2 + \gamma^2 = 8$, then show that :

$$\alpha^3 + \beta^3 + \gamma^3 \geq 16\sqrt{\frac{2}{3}}.$$

(*c*) Prove that if n is a positive integer, then :

(*i*) $\left(1 + \frac{1}{n}\right)^n < \left(1 + \frac{1}{n+1}\right)^{n+1}$

(*ii*) $\left(1 - \frac{1}{n}\right)^n < \left(1 - \frac{1}{n+1}\right)^{n+1}$ 4½,4,4

4. (*a*) Sum to n terms the series

$\cos\theta \cos\theta + \cos^3\theta \cos 3\theta + \cos^5\theta \cos 5\theta + \ldots\ldots$

(*b*) Find the condition such that the roots of the equation $x^3 - px^2 + qx - r = 0$ are in harmonic progression.

(*c*) Show that :

$$2^n \geq 1 + n\, 2^{\frac{n-1}{2}}.$$ 5½,4,3

Section II

5. (*a*) Evaluate the determinant :

$$\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$

(*b*) Show that :

$$\begin{vmatrix} (a-x)^2 & (a-y)^2 & (a-z)^2 \\ (b-x)^2 & (b-y)^2 & (b-z)^2 \\ (c-x)^2 & (c-y)^2 & (c-z)^2 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

(c) Define a circulant determinant with a suitable example. 5,5½,2

6. (a) Prove that every Hermitian matrix H can uniquely be expressed as P + iQ, where P and Q are real symmetric and real skew symmetric matrices respectively. Further show that $H^{\theta}H$ is real if and only if PQ = –QP.

(b) If A, B and C are square matrices of order $n \times n$ and $A = B + C$, then $A^{p+1} = B^{p}[B + (p + 1)C]$, where BC = CB and C^2 is a NULL matrix and p is a positive integer. 6½,6

7. (a) Let $\underset{\sim}{e}$ be the column vector with elements (1, 1, ...1) and $\underset{\sim}{e}'$ its transposed vector. Let A be any square matrix of order n and if we define matrix M as

$$M(x) = I + x\,A\,\underset{\sim}{e}\,\underset{\sim}{e}'$$

where x is a scalar, then

(*i*) Prove that

$$M(x)\ M(y) = M(x + y + \lambda xy)$$

where λ represents sum of all the elements of matrix A.

(*ii*) Find $M^{-1}(x)$.

(*b*) Prove that the inverse of a matrix, if it exists, is unique. 7½,5

8. (*a*) Solve the following set of linear equations :

$$x + ay + a^2z + a^3 = 0$$

$$x + by + b^2z + b^3 = 0$$

$$x + cy + c^2z + c^3 = 0.$$

(*b*) If A and B are matrices such that AB and BA exist, then show that

$$\text{Tr}(AB) = \text{tr}(BA)$$

(*c*) Show that for a non-singular matrix $A_{n \times n}$,

$$|\text{adj}\ (\text{adj}\ A)| = |A|^{(n-1)^2}.$$ 6,3½,3