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Your Roll No. ....

6035

B.Sc. (Hons.)/I Sem. B

STATISTICS—Paper STH-104

(Probability and Statistical Methods—I)

(For admission of 2011 and onwards)

Time : 3 *Hours* *Maximum Marks* : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt *six* questions in all. Question No. 1 is compulsory. Attempt *five* more questions selecting at least *two* from each Section.

1. (*a*) Mean of 100 observations is 50 and S.D. is 10. What will be the new mean and S.D. if 5 is subtracted from each observation and then it is divided by 4 ?

(*b*) Find the C.V. of a frequency distribution given that its mean is 120, mode is 123 and Karl Pearson's coefficient of skewness is –0.3.

(*c*) If $P(A \cup B) = \frac{5}{6}$, $P(A \cap B) = \frac{1}{3}$, $P(\bar{A}) = \frac{1}{2}$, find P(A) and P(B). Also comment on the independence of A and B.

(*d*) If $P(A|C) \geq P(B|C)$ and $P(A|\bar{C}) \geq P(B|\bar{C})$, show that $P(A) \geq P(B)$.

(*e*) If N = 150, (B) = 60, (α) = 80, (AB) = 50, then find Yule's coefficient of association and comment on the independence of attributes A and B.

(*f*) Explain the graphic method of locating the values of median and mode. 2,3,2,3,3,2

Section I

2\. (*a*) Define Pearsonian coefficients β_1, β_2, γ_1, γ_2 and discuss their utility in statistics. Also prove that $\beta_2 \geq 1$.

(*b*) Show that in a discrete series if deviations are small compared with mean M, so that $\left(\frac{x}{M}\right)^3$ and higher powers of $\left(\frac{x}{M}\right)$ are neglected, then :

$$\text{Mean}\left(\sqrt{X}\right) = \sqrt{M}\left(1 - \frac{\sigma^2}{8M^2}\right)$$

where 'σ' is the standard deviation. 6,6

3. (*a*) If for a random variable X, the absolute moment of order $K(\beta_K)$ exists for K = 1, 2, ..., $n - 1$, then show that :

(*i*) $\beta_K^{2K} \leq \beta_{K-1}^K \ \beta_{K+1}^K$

(*ii*) $\beta_K^{1/K} \leq \beta_{K+1}^{1/K+1}$; $K = 1, 2, ..., n - 1$.

(*b*) Explain the term "dispersion". Describe different measures of dispersion, mentioning their merits and demerits.

Let r be the range and S the standard deviation of a set of observations x_1, x_2, ..., x_n. Prove that $S \leq r$. 6,6

4. (*a*) Explain the 'Principle of Least Squares' and describe its application in fitting a curve of the form :

$$y = a \exp(bx + cx^2).$$

(*b*) What do you understand by consistency of given data (in a population containing N items, each of which is characterised by the attributes A, B, C) ? How do you check it ? Given that :

$$(A) = (\alpha) = (B) = (\beta) = (C) = (\gamma) = \frac{1}{2}N$$

and $(ABC) = (\alpha\beta\gamma)$,

show that :

$$2(ABC) = (AB) + (BC) + (AC) - \frac{1}{2}N.$$ 6,6

5. (*a*) Show that :

(*i*) if all A's are B's and all B's are C's, then all A's are C's.

(*ii*) if all A's are B's and no B's are C's, then no A's are C's.

(*b*) Define 'moments'. Obtain the relationship between central moment of order $r = (\mu r)$ in terms of moments about origin $(\mu r')$. Hence, express μ_4 in terms of μ_1', μ_2', μ_3' and μ_4'. 6,6

Section II

6. (*a*) Give the Empirical (statistical) definition of probability and give its advantages over classical definition of probability.

Four tickets marked 00, 01, 10, 11 are placed in a bag. A ticket is drawn at random five times, being replaced each time. Find the probability that the sum of the numbers on tickets thus drawn is 23.

(*b*) A, B and C are three random events and are pairwise independent. Also A is independent of (B ∪ C). Show that A, B and C are mutually independent. 6,6

7. (*a*) State and prove Bayes' theorem.

(*b*) There are 3 persons A, B, C. The probability that A alone will survive for 10 years is 4/105 and the probability that C alone will die within 10 years is 2/21. Assuming that the events of the survival of A, B and C can be regarded as independent, calculate the probability of surviving 10 years for person B. 6,6

8. (*a*) Suppose that there is a chance for a newly constructed building to collapse, whether the design is faulty or not. The chance that the design is faulty is 10%. The chance that the building collases is 95% if the design is faulty and otherwise it is 4%. It is seen that the building collapsed. What is the pobability that it is due to faulty design ?

(*b*) $A_1, A_2, \ldots\ldots, A_n$ are n independent events with

$$P(A_i) = 1 - \frac{1}{\alpha^i}, \; i = 1, 2, \ldots\ldots, n.$$

Find the value of

$$P(A_1 \cup A_2 \cup \ldots\ldots \cup A_n).$$ 6,6