[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8778 **C** **Roll No............**

Unique Paper Code : 237153

Name of the Paper : STHT-103 : Algebra – I

Name of the Course : B.Sc. (H) Statistics, Part I

Semester : I

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Six** questions in all, selecting **three** questions from each Section.

SECTION I

1. (a) Solve the equation $x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$, the roots being in G.P.

   (b) Form the cubic whose roots are the values of α, β, γ given by the relation $\Sigma\alpha = 3$, $\Sigma\alpha^2 = 5$, $\Sigma\alpha^3 = 11$. Hence or otherwise find the values of $\Sigma\alpha^4$ and $\Sigma\alpha^{-4}$. (5½,7)

2. (a) If $a = \cos\alpha + i\sin\alpha$, $b = \cos\beta + i\sin\beta$, $c = \cos\gamma + i\sin\gamma$, and

   $a/b + b/c + c/a = -1$, prove that

   $\cos(\beta - \gamma) + \cos(\gamma - \alpha) + \cos(\alpha - \beta) + 1 = 0$.

   (b) Show that the equation

   $$\cos(2\theta - \alpha) = p\cos(\theta - \beta)$$

   can be satisfied by four values θ_1, θ_2, θ_3, θ_4 of θ, of which no two differ by a multiple of 2π. Show also that

   $$\theta_1 + \theta_2 + \theta_3 + \theta_4 - 2\alpha = 2k\pi, \text{ k being an integer.}$$

(c) Sum to n terms the series : $\sin\theta + \frac{1}{3}\sin 2\theta + \frac{1}{3^2}\sin 3\theta + \ldots$ (4½,4,4)

3. (a) If a, b, c are positive integers, then prove that

$$\left(\frac{bc+ca+ab}{a+b+c}\right)^{a+b+c} > \sqrt{(bc)^a (ca)^b (ab)^c}\ .$$

(b) If x, y, z are all positive numbers such that $x^3 + y^3 + z^3 = 81$, then prove that

$$x + y + z \leq 9.$$

(c) If a, b, c > 0 and $a \neq b \neq c$, then show that

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} > \frac{3}{2}.$$ (4½,4,4)

4. (a) Prove that

$256 \sin^5\theta \cos^4\theta = \sin 9\theta - \sin 7\theta - 4\sin 5\theta + 4\sin 3\theta + 6\sin\theta.$

(b) Solve the equation

$$x^3 + 6x^2 + 12x - 19 = 0$$

by removing its second term.

(c) If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$, then find the value of

(i) $\sum \alpha^2\beta$

(ii) $\sum \alpha^2\beta^2$ (5,4½,3)

SECTION II

5. (a) Show that the possible square roots of the two rowed unit matrix I are

$$\pm I \text{ and } \begin{pmatrix} \alpha & \beta \\ \gamma & -\alpha \end{pmatrix} \text{ where } 1 - \alpha^2 = \beta\gamma\ .$$

(b) Prove that

(i) $tr(AA') \geq o$,

(ii) If $A = (a_{ij})$ is a symmetric matrix of order n, then

$$tr(A^2) = \sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}^2 .$$

(c) If A is a symmetric and B a skew-symmetric matrix, both of order n such that (A + B) is non-singular and $C = (A + B)^{-1}(A - B)$, then prove that

(i) $C'(A+B)C = A + B$

(ii) $C'(A-B)C = A - B$

(iii) $C'AC = A$ (3½,5,4)

6. (a) If A is a non-singular matrix of order n then

(i) $|adj\,A| = |A|^{n-1}$

(ii) $adj(adj A) = |A|^{n-2} A$

(b) For each real number x such that $-1 < x < 1$, let A(x) be the matrix defined as $A(x) = (1-x^2)^{-1/2}\begin{pmatrix} 1 & -x \\ -x & 1 \end{pmatrix}$. Show that $A(x)\,A(y) = A(z)$, where $z = \frac{x+y}{1+xy}$. Deduce that $(A(x))^{-1} = A(-x)$.

(c) Prove that if the matrix product AB of two square matrices is zero then either A = O or B = O or both A and B are singular matrices. (4,4½,4)

7. (a) Evaluate

$$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^3 & b^3 & c^3 \end{vmatrix}.$$

Hence find its value when $a + b + c = 0$.

(b) Prove that every square matrix can be expressed uniquely as P + iQ where P and Q are Hermitian matrices.

(c) Define Nilpotent matrix. If A is nilpotent matrix of index 2, then show that

$A(I \pm A)^n = A.$ (4,5,3½)

8. (a) Solve the following equations :

$$x + 2y + 3z = 14$$
$$3x + y + 2z = 11$$
$$2x + 3y + z = 11$$

with the help of matrices.

(b) Prove that

$$\begin{vmatrix} a+ib & c+id \\ -c+id & a-ib \end{vmatrix} \times \begin{vmatrix} \alpha-i\beta & \gamma-i\delta \\ -\gamma-i\delta & \alpha+i\beta \end{vmatrix}$$

where $i^2 = -1$, can be written in the form

$$\begin{vmatrix} A-iB & C-iD \\ -C-iD & A+iB \end{vmatrix}$$

and hence prove that

$$(a^2 + b^2 + c^2 + d^2)(\alpha^2 + \beta^2 + \gamma^2 + \delta^2) = (A^2 + B^2 + C^2 + D^2).$$

(c) Show that a skew symmetric determinant of odd order vanishes.

(4,5½,3)