[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8777 **C** **Roll No............**

Unique Paper Code : 237152

Name of the Paper : STHT-102 : Calculus – I

Name of the Course : B.Sc. (Hons.) Statistics, Part I

Semester : I

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **six** questions in all.
3. Question No. **1** is compulsory and select **three** questions from **Section–I** and **two** questions from **Section–II**.

1. (a) Discuss the differentiability of the function :

$$f(x)=\begin{cases}\dfrac{x\left(e^{\frac{1}{x}}-1\right)}{e^{\frac{1}{x}}+1}, & x\neq 0\\ 0, & x=0.\end{cases}$$

at x = 0.

(b) If $u=\tan^{-1}x$, show that $(1+x^2)\dfrac{d^2u}{dx^2}+2x\dfrac{du}{dx}=0$. Hence find $\left(\dfrac{\partial^2 u}{\partial x^2}\right)_{x=0}$.

(c) If $u=\sin^{-1}\left(\dfrac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)$, show that $x\dfrac{\partial u}{\partial x}+y\dfrac{\partial u}{\partial y}=0$.

(d) Find the asymptotes parallel to axis to curve $xy = 2$.

(e) Find the particular integral of differential equation $\left(D^2+2D+1\right)y=x^2$.

(3×5=15)

SECTION I

2. (a) If $y=\sin\left(m\sin^{-1}x\right)$, show that

$(1-x^2)y_{n+2}=(2n+1)x\,y_{n+1}+\left(n^2-m^2\right)y_n$ and hence find $y_n(0)$.

(b) If $u=x\left(1-r^2\right)^{-\frac{1}{2}}, v=y\left(1-r^2\right)^{-\frac{1}{2}}$ and $w=z\left(1-r^2\right)^{-\frac{1}{2}}$, where

$r^2=x^2+y^2+z^2$, prove that $J(u,v,w)=\left(1-r^2\right)^{-\frac{5}{2}}$. (6,6)

3. (a) Find the points of inflexion of the curve $y=(\log x)^3$.

(b) Find all the asymptotes to the curve

$4\left(x^4+y^4\right)-17x^2y^2-4x\left(4y^2-x^2\right)+2\left(x^2-2\right)=0$

and show that they pass through points of intersection of the curve with the ellipse $x^2+4y^2=4$. (6,6)

4. (a) Locate the double points of the curve $y^2=(x-1)(x-2)^2$, and discuss their nature.

(b) Trace the curve $y^2=\dfrac{x^2(x+a)}{x-a}$. (6,6)

5. (a) Trace the curve $r=\dfrac{3a\sin\theta\cos\theta}{\cos^3\theta+\sin^3\theta}$.

(b) If $w=\log\left(x^2+y^2+z^2\right)$, show that

$\left(x^2+y^2+z^2\right)\left(\dfrac{\partial^2 w}{\partial x^2}+\dfrac{\partial^2 w}{\partial y^2}+\dfrac{\partial^2 w}{\partial z^2}\right)=1.$ (6,6)

6. (a) Discuss maximum or minimum value of the function $u = x^3 + y^3 - 3axy$.

(b) If $u = \sin^{-1}\left\{\frac{x^{\frac{1}{3}} + y^{\frac{1}{3}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}\right\}^{\frac{1}{2}}$, show that

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2\frac{\partial^2 u}{\partial y^2} = \frac{\tan u}{144}\left(13 + \tan^2 u\right).$$ (6,6)

SECTION II

7. (a) By using the transformation $x^2 = u$ and $y^2 = v$, solve

$(px - y)(py + x) = h^2 p$, where $p = \frac{dy}{dx}$.

(b) Solve $\frac{dy}{dx} + x\sin 2y = x^3\cos^2 y$.

(c) Solve $x^2 y dx - (x^3 + y^3)dy = 0$. (4,4,4)

8. (a) Show that the equation $\left(1 + e^{\frac{x}{y}}\right)dx + e^{\frac{x}{y}}\left(1 - \frac{x}{y}\right)dy = 0$ is exact and hence find its solution.

(b) Solve differential equation of first order

(i) $p^2 + 2py\cot x = y^2$

(ii) $y = 2px + p^n$

Where, $p = \frac{dy}{dx}$. (4,8)

9. Solve any **two** of the following differential equations :

(i) $\frac{d^3y}{dx^3}+\frac{d^2y}{dx^2}-\frac{dy}{dx}-y=\cos 2x$

(ii) $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=x^2+e^x+\cos x$

(iii) $x^3\frac{d^3y}{dx^3}+2x^2\frac{d^2y}{dx^2}+2y=10\left(x+\frac{1}{x}\right)$ (6,6)