[This question paper contains 4 printed pages.]

Sr. No. of Question Paper : 8779 **C** **Roll No............**

Unique Paper Code : 237101

Name of the Paper : STHT-104 : Probability & Statistical Methods – I

Name of the Course : B.Sc. (Hons.) Statistics, Part I

Semester : I

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on receipt of this question paper.
2. Attempt **Six** questions in all.
3. Question No. **1** is compulsory.
4. Attempt **five** more questions, selecting at least **two** from each section.

1. (a) Fill in the blanks :–

   (i) Minimum value of root mean square deviations is ________ .

   (ii) If $\beta_2 > 3$, the distribution is said to be ________ .

   (iii) Attributes A and B are said to be independent if ________ .

   (iv) The mode and median of 100 items are 50 and 52 respectively. The value of the largest item is 100. It was later found that it was actually 110. Then the true median is ________ .

   (v) If $P(A) = P(A|B) = 1/4$ and $P(B|A) = 1/2$. Then $P(\bar{A}|B) =$ ________ .

   (b) (i) Compute the geometric mean of 2, 4, 16 and 32.

   (ii) Find whether the attributes α and β are associated or not. Given (AB) = 500; (α) = 800; (β) = 600 and N = 1500.

   (iii) It is known that $P(A) = 3/4$ and $P(B) = 2/3$. Show that $\frac{5}{12} \le P(A \cap B) \le \frac{2}{3}$.

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(iv) The first three moments about the origin are given by

$$\mu_1' = \frac{n+1}{2};\ \mu_2' = \frac{(n+1)(2n+1)}{6};\ \mu_3' = \frac{n(n+1)^2}{4}$$

Examine the skewness of the data.

(v) If A and B are two mutually exclusive events, then show that

$$P\left(A|\bar{B}\right) = \frac{P(A)}{\left[1-P(B)\right]}.$$ (5,2,2,2,2,2)

SECTION I

2. (a) Show that in a discrete series if deviations $x_i = X_i - M$, i = 1, 2, ..., n are small compared with mean M so that $\left(\frac{x_i}{M}\right)^3$ and higher powers of $\left(\frac{x_i}{M}\right)$ are neglected; then –

(i) $G = M\left(1-\frac{\sigma^2}{2M^2}\right)$

(ii) $H = M\left(1-\frac{\sigma^2}{M^2}\right)$

where G is the Geometric Mean, H is the Harmonic mean and σ is the standard deviation of the distribution.

(b) Discuss the principle of least squares. Derive the normal equations for fitting the curve $Y = ae^{bX}$ to the given set of n points $\{(x_i, y_i);\ i = 1, 2, \ldots, n\}$.

(6,6)

3. (a) A frequency distribution is divided into two parts. The mean and standard deviation of the first part are $\bar{x}_1$ and σ_1 and those of the second part are $\bar{x}_2$ and σ_3 respectively. Obtain the mean $\bar{x}$ and standard deviation σ of the combined distribution.

(b) If $\delta = (AB) - (AB)_0$ then with the usual notation prove that –

(i) $\left[(A)-(\alpha)\right]\left[(B)-(\beta)\right]+2N\delta = (AB)^2+(\alpha\beta)^2-(A\beta)^2-(\alpha B)^2$

(ii) $\delta = \frac{(A)(\alpha)}{N}\left[\frac{(AB)}{(A)} - \frac{(\alpha\beta)}{(\alpha)}\right]$ (6,6)

4. (a) From a sample of n observations the A.M. and variance are calculated. It is then found that one of the values x_1 is in error and should be replaced by x_1'. Show that the aadjustment to the variance to correct the error is $\frac{1}{n}\left(x_1' - x_1\right)\left(x_1' + x_1 - \frac{x_1' - x_1 + 2T}{n}\right)$ where T is the total of the original results.

(b) Among the population of a certain city 50% are males, 60% are wage earners and 50% are 45 years of age or above, 10% of males are not wage earners and 40% of the males are under 45. Make the best possible inference about the limits within which the percentage of people (male or female) of 45 years or above are wage earners. (6,6)

5. (a) Prove that for any discrete distribution, standard deviation is not less than mean deviation from mean.

(b) For a random variable X moments of all orders exist. Show that –

(i) $\mu_{2j+1}^2 \le \mu_{2j}\mu_{2j+2}$

(ii) $\mu_j^{1/j} \le \mu_{j+1}^{1/(j+1)}$

where μ_j is the j^{th} central moment. (5,7)

SECTION II

6. (a) State and prove both the Boole's inequality.

(b) Three newspapers A, B and C are published in a certain city. It is estimated from a survey that 20% read A, 16% read B, 14% read C, 7% read A and B, 5% read A and C, 4% read B and C and 2% read all the three newspapers. What is the probability that a randomly chosen person –

(i) does not read any paper

(ii) does not read C

(iii) reads A but not B

(iv) reads only one these papers (6,6)

7. (a) A doctor is to visit the patient and from past experience it is known that the probabilities that he will come by train, bus or scooter are respectively $\frac{3}{10}, \frac{1}{5}$ and $\frac{1}{10}$; the probability that he will use some other mode of transport being $\frac{2}{5}$. If he comes by train, the probability that he will be late is $\frac{1}{4}$, if by bus $\frac{1}{3}$ and if by scooter $\frac{1}{12}$ and is uses some other means of transport it can be assumed that he will not be late. When he arrives he is late. What is the probability that he came by bus ?

(b) If A, B and C are three ordinary events and

$$S_1 = P(A) + P(B) + P(C)$$
$$S_2 = P(A \cap B) + P(A \cap C) + P(B \cap C)$$
$$S_3 = P(A \cap B \cap C)$$

Prove that the probability that exactly one of the three events occurs is given by

$$S_1 - 2S_2 + 3S_3$$ (6,6)

8. (a) Two fair dice are thrown independently. Three events A, B and C are defined as follows :

A: Even face with first dice

B: Even face with second dice

C: Sum of the points on the two dice is odd.

Discuss the independence of events A, B and C.

(b) It is known that 40% of the students in a college are girls and 50% of the students are above the median height. If $\frac{2}{3}$ of the boys are above the median height, what is the probability that a randomly selected student who is below the median height is a girl. (6,6)